The fragility of opinion formation in a complex world

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Inselspital/UniBE, Bern

Colloquium, Department of Physics, University of Fribourg
23 February 2022

A motivating experience

In 2014, a series of protests and political demonstrations began in Venezuela...

Reuters

A motivating experience

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Question #1:
Should we trust
Maduro's government?

A motivating experience

In 2014, a series of protests and political demonstrations began in Venezuela...

"Venezuela's government should address the people's legitimate grievances..."

"We must respect the right to peaceful protest..."

"We trust that the government of President Maduro will preserve the constitutional order..."
President of Syria Bashar al-Assad expressed his support in a letter to President Maduro,...

https://en.wikipedia.org/wiki/Reactions_to_the_2014-2017_Venezuelan_protests

A motivating experience

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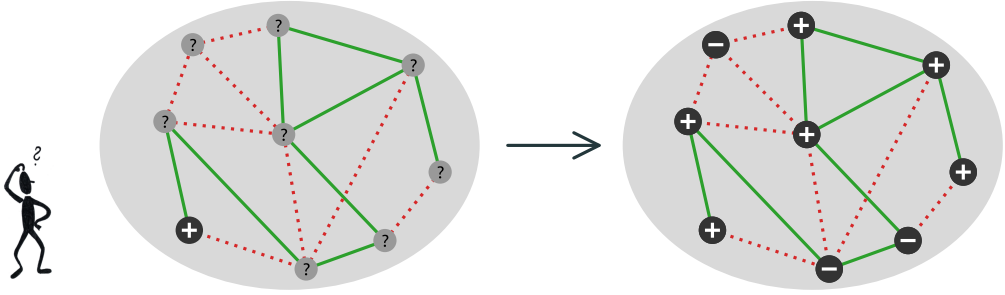
Question #2:
What happens if we generalize
from this single question
to a whole learning process?

Opinion formation on a signed network



- Nodes: Subjects on which opinions are to be made
  - Countries and other entities in world politics
- Links: Signed relations between the subjects
- The observer: Outside, no social network

Social influence in opinion formation

REVIEWS OF MODERN PHYSICS, VOLUME 81, APRIL–JUNE 2009

Statistical physics of social dynamics

Claudio Castellano*

Santo Fortunato†

Vittorio Loreto‡

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Social physics

Marko Jusup [a], Petter Holme [a], Kiyoshi Kanazawa [b,c], Misako Takayasu [a], Ivan Romić [a,d], Zhen Wang [e], Sunčana Geček [f], Tomislav Lipić [g], Boris Podobnik [h,i,j,k], Lin Wang [l], Wei Luo [m], Tin Klanjšček [f], Jingfang Fan [n,o], Stefano Boccaletti [p,q,r], Matjaž Perc [s,t,u,*]

Social influence in opinion formation

- One example for all: The Deffuant model (2000)
  1. Individuals have continuous opinions $x_i \in [0, 1]$, initially uniformly random
  2. They meet by chance, discuss only if $|x_i - x_j| < \delta$
  3. Upon a discussion, their opinions get closer at a convergence rate μ

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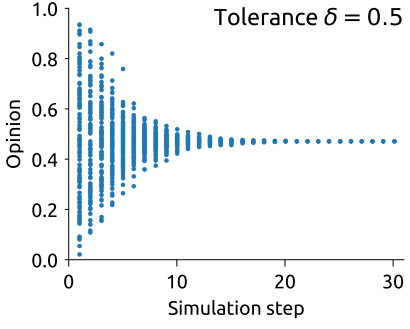

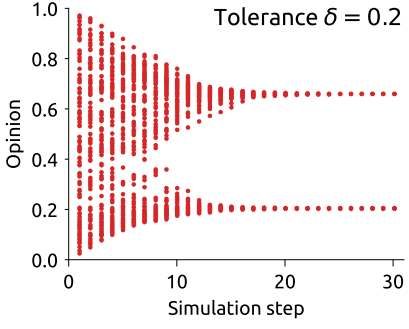

Back to our model: Setting up a synthetic world

- N nodes of two types: $\theta_i \in \{-1, +1\}$; unknown to the observer
- N_S source nodes: The observer knows their types
- Link signs correlate with node types:
  - $\theta_i = \theta_j$: link is positive with probability $r \geq 0.5$
  - $\theta_i \neq \theta_j$: link is negative with probability $r \geq 0.5$
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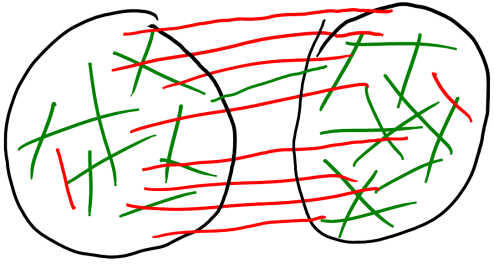

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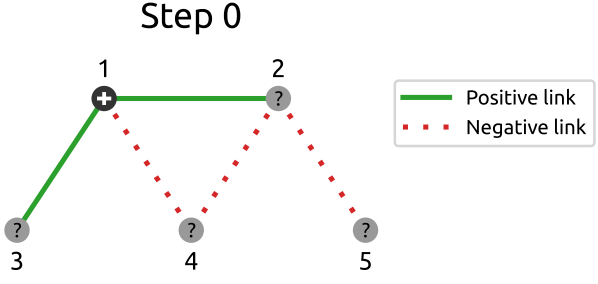
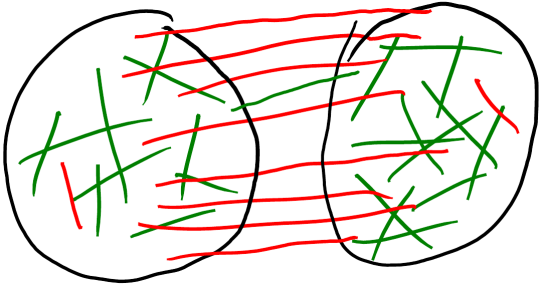


Possible applications of the model

1. Two opposing camps: Mainstream media and misinformation sources
   - You initially trust in some mainstream media
   - Do you end up trusting other mainstream media and distrusting misinformation sources?



Possible applications of the model

1. Two opposing camps: Mainstream media and misinformation sources
2. Employee network: Manager attempts to assess employee qualities
3. Inter-firm network: Which other firms to trust
4. Social networks: E.g., find a suitable roommate
5. ...

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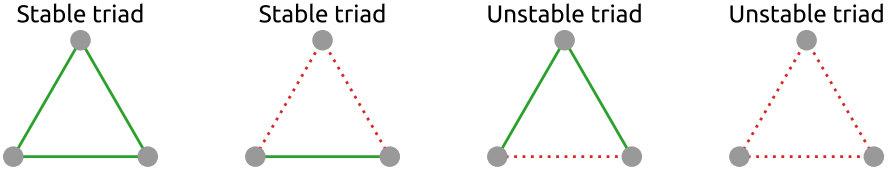
Closely related: Social balance theory (Heider, 1946)



Random neighbor heuristic (Medo et al, 2021)

Local & easy to apply
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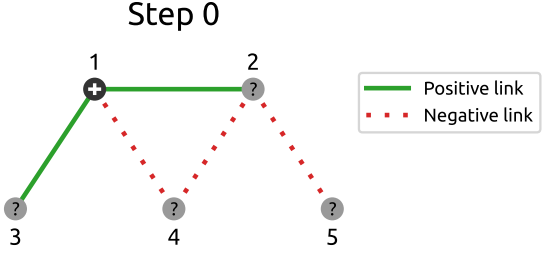

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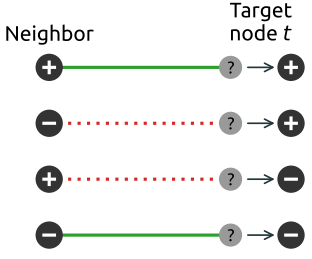

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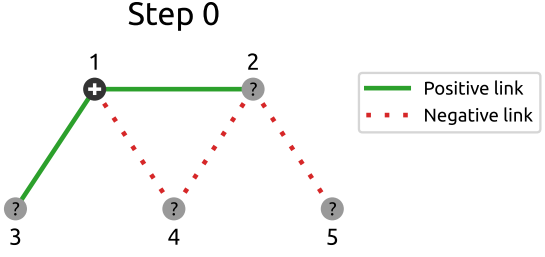

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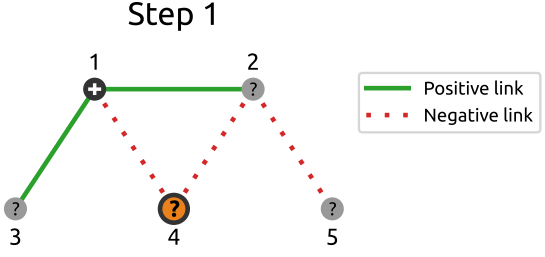

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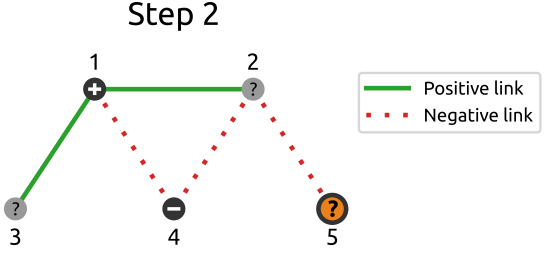

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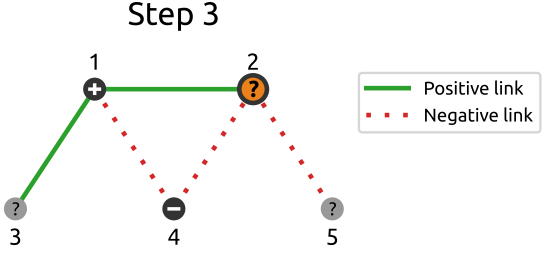

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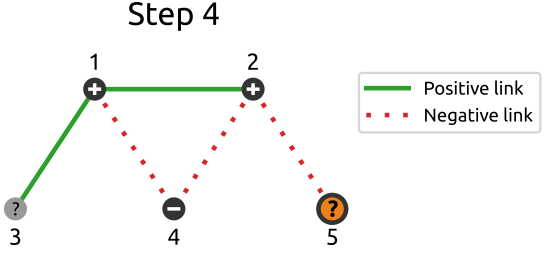
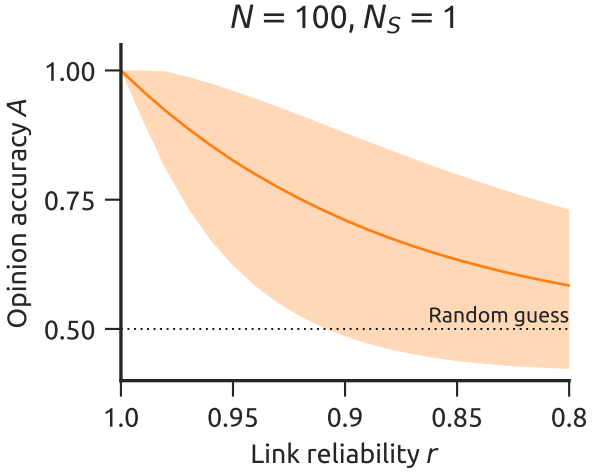


Random neighbor heuristic: The outcome



Shaded area: 10th–90th percentile range

Random neighbor heuristic: The outcome

- Introduce the probability that c out of n opinions are correct, $P(c;n)$

$$P(c;n) = P(c-1;n-1)\,\frac{c(2r-1)+(1-r)(n+1)-1}{n-1} + P(c;n-1)\left[r - \frac{c(2r-1)}{n-1}\right]$$

$$P(c = N_S; n = N_S) = 1$$

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$$E(A) - 1/2 \sim N^{-2(1-r)}$$

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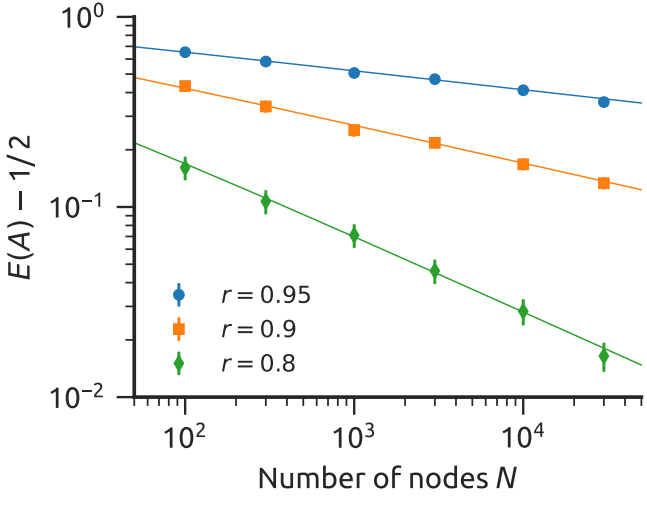


Lesson #1

Even at small noise,
resulting opinions show
low accuracy and high variability

Lesson #2

As the system size grows,
limit opinion accuracy is 1/2
regardless of how small is the noise

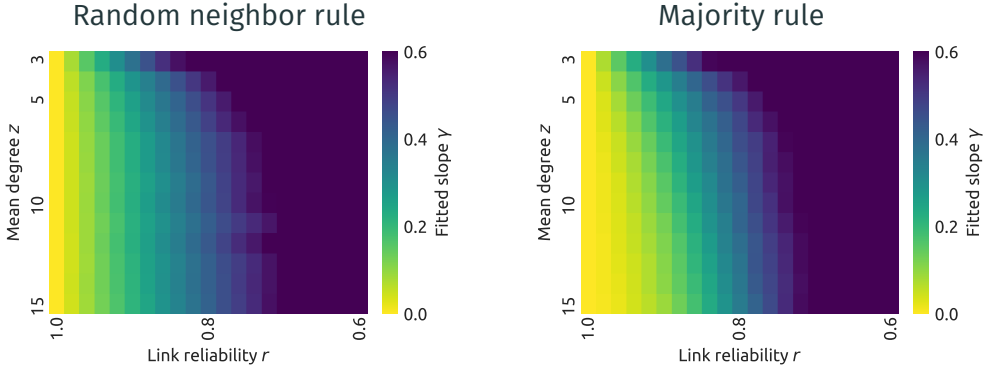
To make sense
of a complex world
is difficult

The majority rule

- Use all neighbors, not just a random one
- Choose the majority opinion signal

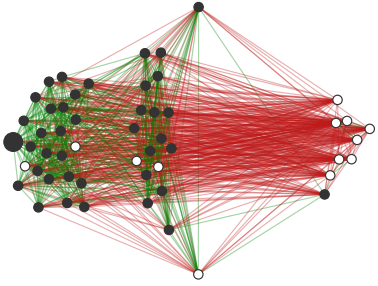
The majority rule

- Use all neighbors, not just a random one
- Choose the majority opinion signal
- Opinion accuracy still approaches 1/2 as $N^{-\gamma}$

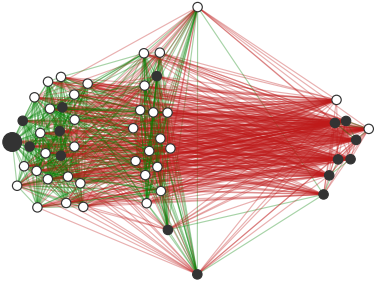


1st United Nations General Assembly network

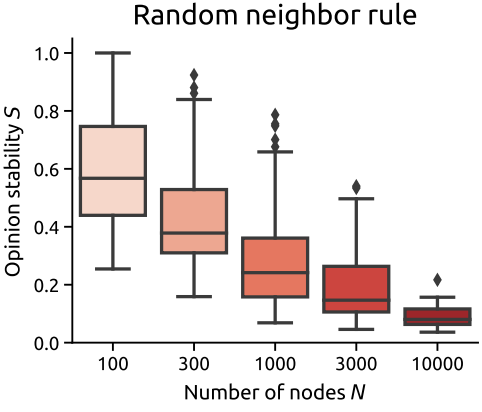
Run 1

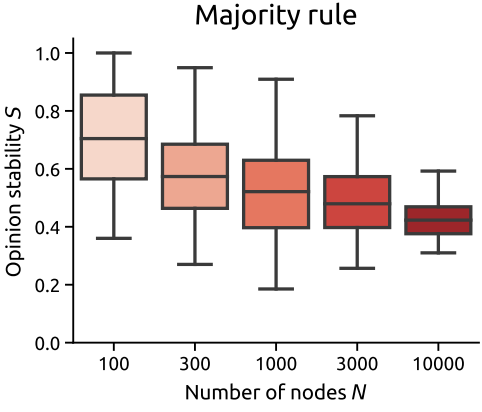


Run 2



Opinion formation on real signed networks





Slashdot social network

Two new methods (Meng et al, 2022)

1. Bayesian solution:

$$P[\boldsymbol{\theta}|\boldsymbol{\sigma}, R] = \frac{P[\boldsymbol{\sigma}, R|\boldsymbol{\theta}] \cdot P[\boldsymbol{\theta}]}{P[\boldsymbol{\sigma}, R]} = \frac{q^{Z_1(\boldsymbol{\theta})}(1-q)^{Z_2(\boldsymbol{\theta})} r^{Z_3(\boldsymbol{\theta})}(1-r)^{Z_4(\boldsymbol{\theta})}}{\sum_{\boldsymbol{\theta}' \in \Theta} q^{Z_1(\boldsymbol{\theta}')}(1-q)^{Z_2(\boldsymbol{\theta}')} r^{Z_3(\boldsymbol{\theta}')}(1-r)^{Z_4(\boldsymbol{\theta}')}}$$

2. Shortest-path heuristic:
   - Based on shortest paths between each source node and a target node
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Theorem (Ordering)

For a given network, set of source nodes S and target node t, the expected accuracies of the three rules are be ordered as

$$E[A^{Bayes}] \geq E[A^{ShPath}] \geq E[A^{RNeighbor}].$$

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Shortest-path accuracy: $E(A) - 1/2 \sim N^{-\gamma}$ where

$$\gamma = -\ln(2r-1)/\ln z$$

on a random network

How to avoid
ending up with
random opinions?

Option 1: Start with many source nodes

- More source nodes $\implies$ better accuracy
- Denoting $f_S := N_S/N$, the random neighbor rule gives

$$\lim_{N\to\infty} E(A) = \frac{1}{2} + f_S^{2(1-r)}/2$$

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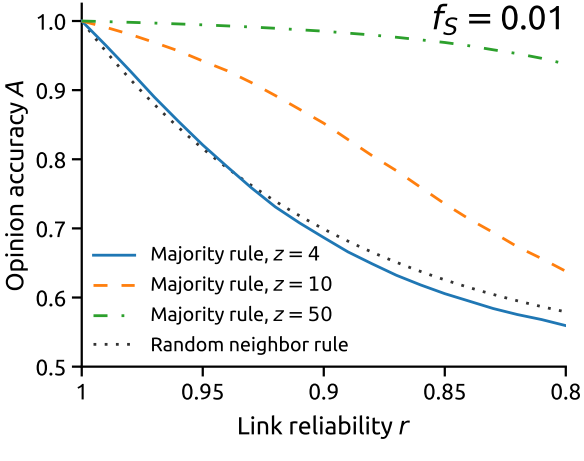

Option 2: Require consensus (Fenoaltea et al, 2022)

- Rephrase from opinion formation to group growth:
  1. Individuals of two types: Fit or unfit for a group
  2. The group initially consists of N_0 fit members
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- Practical difference:
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- Why group cohesion matters:
  - Cohesive groups perform better and exist longer

Group cohesion: Results

- When $m = 1$, cohesion goes to $1/2$ as $N^{-2(1-r)}$

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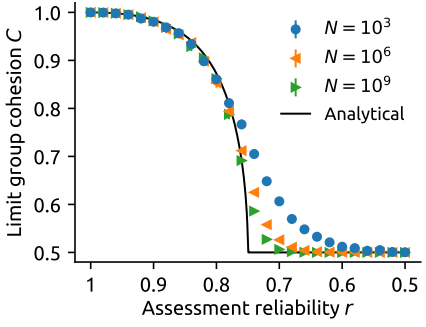

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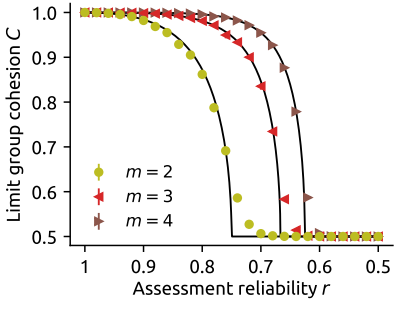
- For general m, the critical point is $r_c = 1/2 + 1/(2m)$



Broader implications

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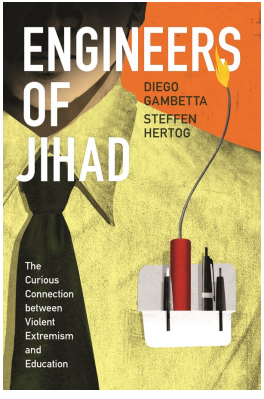

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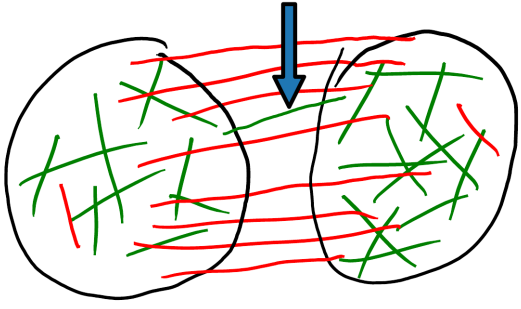

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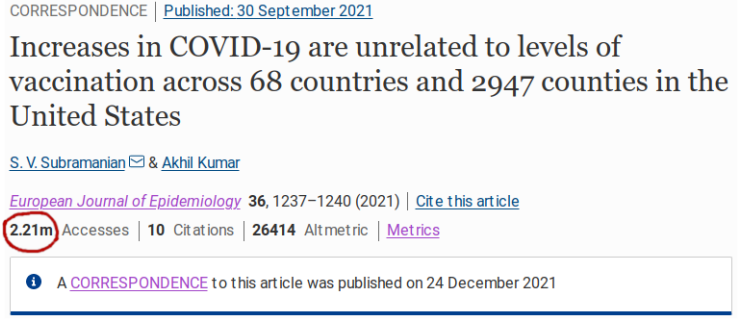

CORRESPONDENCE | Published: 30 September 2021

Increases in COVID-19 are unrelated to levels of vaccination across 68 countries and 2947 counties in the United States

S. V. Subramanian & Akhil Kumar

European Journal of Epidemiology **36**, 1237–1240 (2021) | Cite this article

2.21m Accesses | **10** Citations | **26414** Altmetric | Metrics

A CORRESPONDENCE to this article was published on 24 December 2021

Instead of acknowledgements

CARITAS Schweiz Suisse Svizzera Svizra
Das Richtige tun


Terre des hommes
Kinderhilfe weltweit.

MEDECINS SANS FRONTIERES
ÄRZTE OHNE GRENZEN

HEKS
Brot für alle.

Thank you for your attention!

M. Medo, M. S. Mariani, L. Lü, The fragility of opinion formation in a complex world, Communications Physics 4, 1 (2021)
F. Meng, M. Medo, B. Buechel, Whom to Trust in a Signed Network? Optimal Solution and two Heuristic Rules, preprint (2022)
E. M. Fenoaltea, F. Meng, R.-R. Liu, M. Medo, Robustness of cohesion in a model of group formation, preprint (2022)

matus.medo@unifr.ch