Self-organized model of cascade spreading

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Econophysics Colloquium
(ECCS'11 Satellite Meeting)

September 15, 2011

State of the art

- Cascades occur in economic systems
  - Stock prices suddenly dropping in a market crash
  - Companies going bankrupt simultaneously
- Multitude of theoretical models
  - Herding behavior of traders (Cont & Bouchaud, 2000)
  - Shortage and bankruptcy propagation in production networks (Weisbuch & Battiston, 2007)
  - Default propagation in credit networks (Sieczka & Hołyst, 2009)
  - Interaction of firms through a bank (Iyetomi *et al*, 2009)
  - Complex credit network economy (Delli Gatti *et al*, 2009)

Motivation

- Search for a minimal model
  - Ignoring causes and details of contagion
  - Allowing for an analytical solution

- Turcotte, 1999:

  “Stock markets expand and grow on relatively long time scales but contract in stock-market crashes on relatively short time scales.”

- Sornette, 2006:

  “Stock crashes (are) caused by the slow buildup of long-range correlation leading to a global cooperative behavior of the market eventually ending into a collapse in a short time interval.”

Basic model

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  - It measures how the node reacts to failures of its neighbors
- At time step t:
  1. One failed node (“trigger”) is chosen at random
  2. If a neighbor of node i fails, node i fails too with probability f_i
  3. The cascade of failures propagates until all remaining nodes resist
  4. Fragility values are updated as

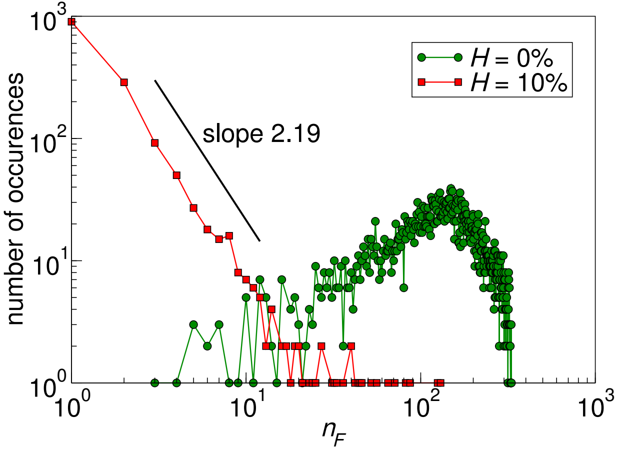
$$f_i(t+1) = \begin{cases} \lambda f_i(t) & i \in \mathcal{F} \\ (1+\beta) f_i(t) & i \in \mathcal{H} \end{cases}$$

where $0 < \beta \ll 1$ and $\lambda \in (0, 1)$

Empirical observations

- Investigate co-occurring price movements of real stocks
- Stock fails when its relative price loss exceeds threshold H
- $n_F(t)$: number of stocks failing at time $t \iff$ cascade size
- Input data
  - Daily closing prices of stocks from the S&P 500 index
  - 332 companies that are quoted from 1992 until May 2010
  - Source: `finance.yahoo.com`

Empirical cascade size distribution



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  - Repeated failures: $P(F_{t+1}|F_t) \gg P(F)$
- Majority of stocks become more resistant after a failure

$$P_i(H_{t+1}|F_t) > P_i(H) \quad \text{and} \quad P_i(F_{t+1}|F_t) < P_i(F)$$

  - Despite volatility clustering!

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Failure probability P_F: $\langle f(t_{\mathrm{eq}} + T)\rangle = \langle f(t_{\mathrm{eq}})\rangle \lambda^{P_F T}(1+\beta)^{(1-P_F)T}$

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A single failure has $N\langle f \rangle$ descendants on average:

$$\langle S \rangle = \frac{1}{1 - N\langle f \rangle} \implies \langle f \rangle = \frac{1}{N}\left(1 - \frac{\ln\left[(1+\beta)/\lambda\right]}{N \ln(1+\beta)}\right)$$

Cascade size distribution

- Branching process approximation:

$$P(S|\beta,\lambda) = \frac{1}{S}\binom{NS}{S-1}\langle f\rangle^{S-1}\big(1-\langle f\rangle\big)^{NS-S+1}$$

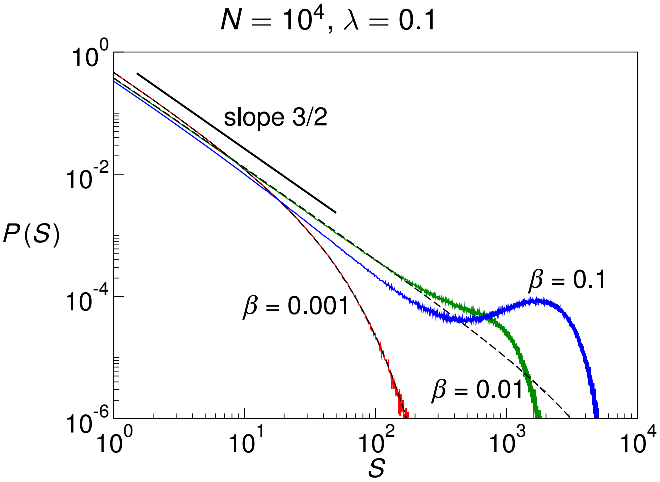
- When $1 \ll S \ll N$:

$$P(S|\beta,\lambda) = \frac{(N\langle f\rangle)^{S-1}\mathrm{e}^{S(1-N\langle f\rangle)}}{\sqrt{2\pi}S^{3/2}}$$

- In the limit $N \to \infty$:

$$P(S|\beta,\lambda) \sim S^{-3/2}$$

Numerical results



Generalizations

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2. Introducing "partial memory"
   - With probability α, a failed node remains failed in the next step (It acts as an additional initial failed node)
   - This brings volatility clustering to the game
   - When $\alpha = 0.04$:

$$\text{model: } P(F|F) = 0.041, P(F) = 0.004, P(F|N) = 0.004$$
$$\text{empirical: } P(F|F) = 0.039, P(F) = 0.003, P(F|N) = 0.003$$

Thank you for your attention