Vektorová funkcia skalárneho argumentu

Hodograf - množina koncových bodov vektora

Ak ku každému skaláru t možno priradiť vektor **A** z určitej oblasti vektorového priestoru, hovoríme, že **A** je vektorovou funkciou skalárneho argumentu

S vektorovými funkciami možno uskutočniť možno uskutočniť rovnaké matematické operácie ako so skalárnymi funkciami

$$\vec{A} = A_x(t)\vec{i} + A_y(t)\vec{j} + A_z(t)\vec{k}$$

Konce vektora popisujú krivku $[A_x(t), A_y(t), A_z(t)]$ - hodograf

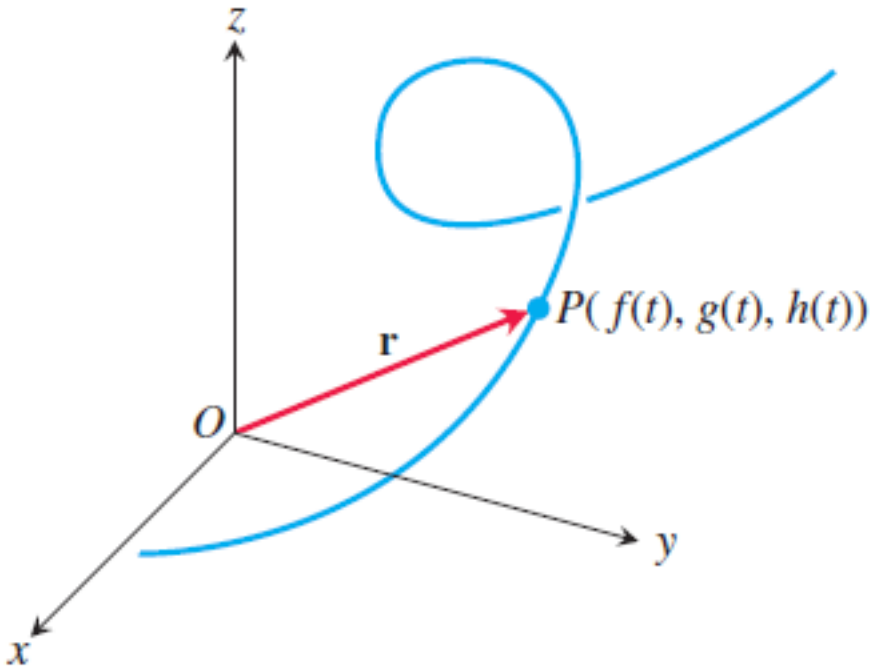
Vektorová funkcia skalárneho argumentu

Častá vektorová funkcia vo fyzike, kinematike

A=r, hodograf je trajektória bodu

$$\vec{r} = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

Skalárny parameter : φ,t,s....



Skalárny parameter

Rôzne parametrizácie kružnicového oblúka

$$\mathbf{r}_1(t) = \sin t\,\mathbf{i} + \cos t\,\mathbf{j}, \qquad (-\pi/2 \le t \le \pi/2),$$

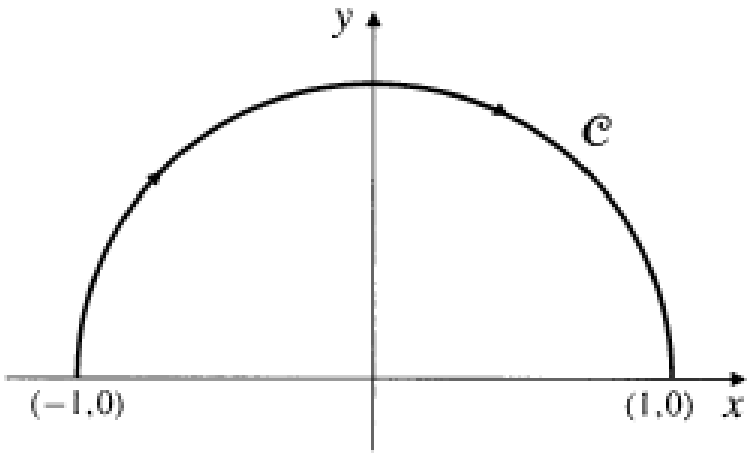
$$\mathbf{r}_2(t) = (t-1)\,\mathbf{i} + \sqrt{2t - t^2}\,\mathbf{j}, \qquad (0 \le t \le 2),$$

$$\mathbf{r}_3(t) = t\sqrt{2 - t^2}\,\mathbf{i} + (1 - t^2)\,\mathbf{j}, \qquad (-1 \le t \le 1)$$

Velkosť sa nemení

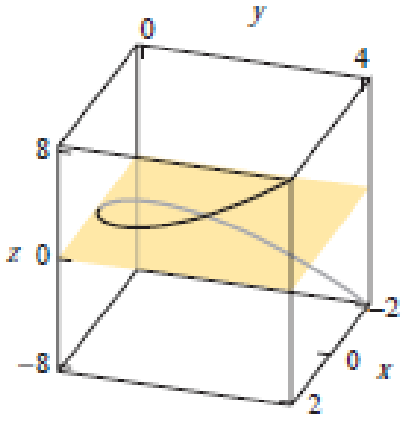
$$\mathbf{r}_2(0) = -\mathbf{i}, \quad \mathbf{r}_2(2) = \mathbf{i}, \quad |\mathbf{r}_2(t)| = \sqrt{(t-1)^2 + 2t - t^2} = 1,$$

$$\mathbf{r}_3(-1) = -\mathbf{i}, \quad \mathbf{r}_3(1) = \mathbf{i}, \quad |\mathbf{r}_3(t)| = \sqrt{t^2(2 - t^2) + (1 - t^2)^2} = 1.$$

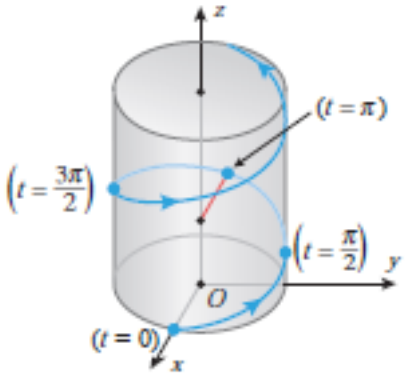


Vektorová funkcia skalárneho argumentu

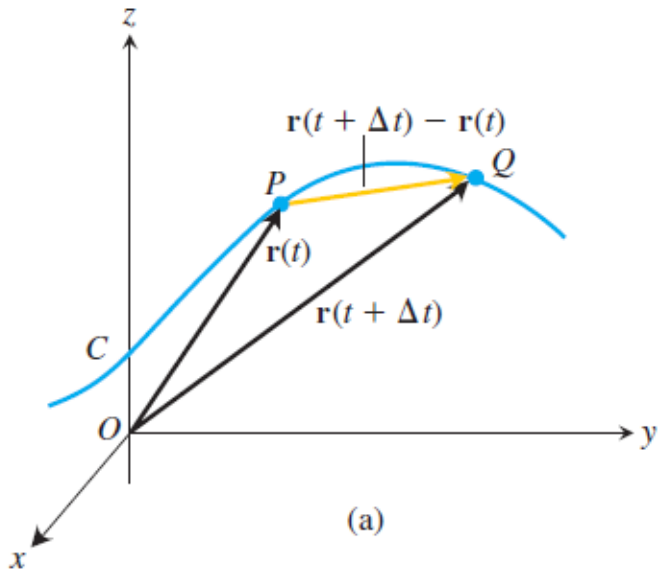
Hodograf - množina koncových bodov vektora



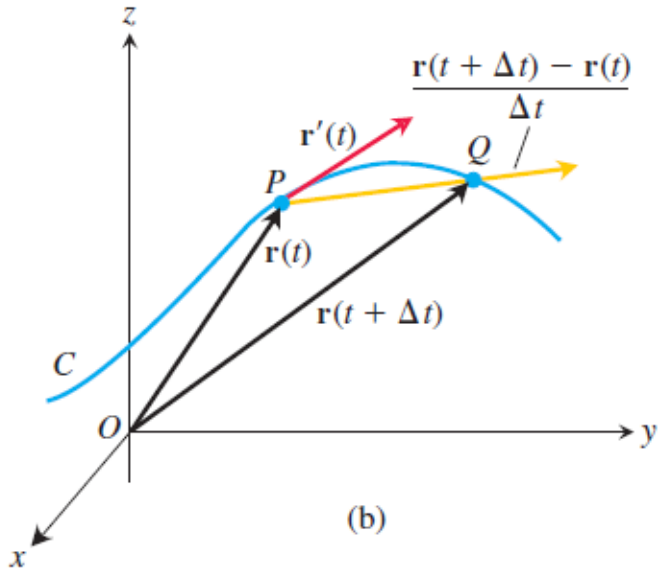
$$\vec{r} = (x, y, z) = (t, t^2, t^3) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$$



$$\vec{r} = (x, y, z) = (a\cos t, a\sin t, ct) = a\cos t\vec{i} + a\sin t\vec{j} + ct\vec{k}$$



(a)

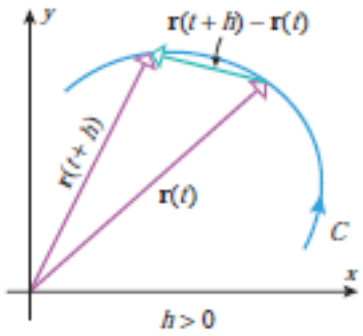


(b)

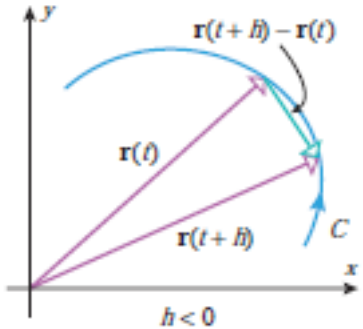
$$\vec{r}' = \frac{d\vec{r}}{dt} = \lim_{\Delta t \to 0} \frac{\vec{r}\left(t+\Delta t\right) - \vec{r}\left(t\right)}{\Delta t}$$

Derivovaný vektor $\vec{r}'$ má smer dotyčnice ku krivke C a je orientovaný v tom smere, v ktorom sa zväčšuje parameter t

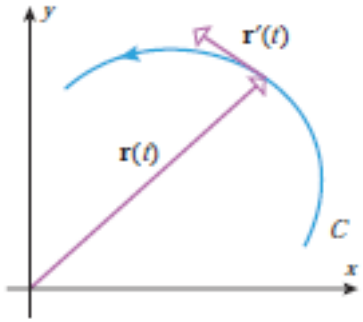
Derivovanie vektorovej funkcie podľa parametra



(a)



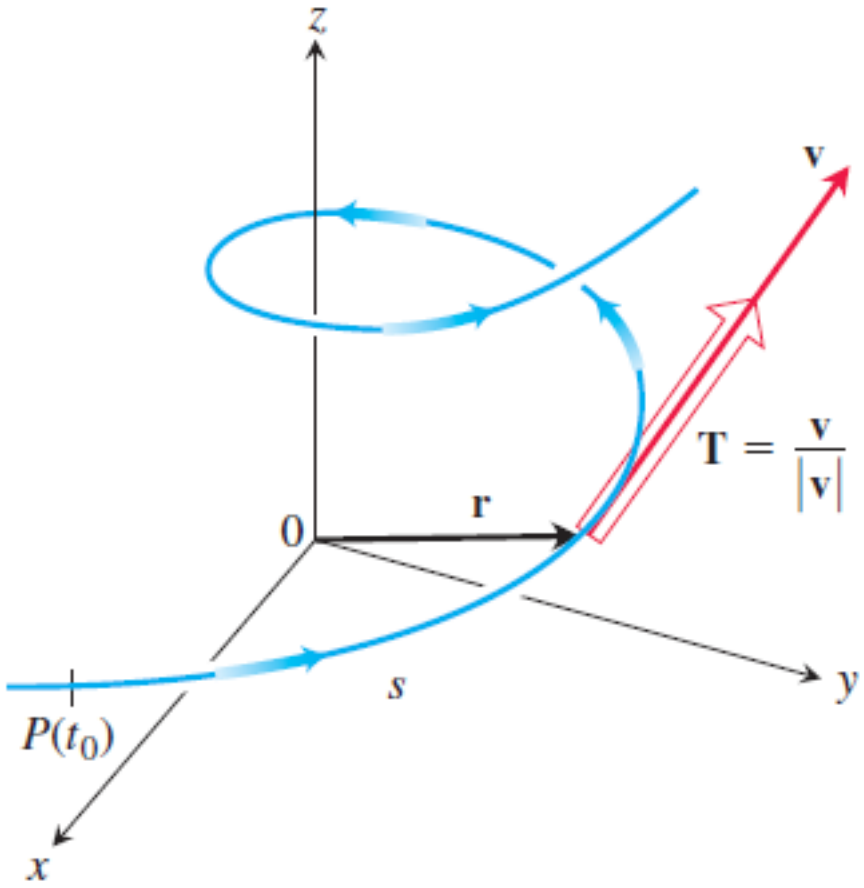
(b)



(c)

$$\vec{r}' = \frac{d\vec{r}}{dt} = \lim_{\Delta t \to 0} \frac{\vec{r}\left(t + \Delta t\right) - \vec{r}\left(t\right)}{\Delta t}$$

Derivovaný vektor $\vec{r}'$ má smer dotyčnice ku krivke C a je orientovaný v tom smere, v ktorom sa zväčšuje parameter t



Vektor rýchlosti je dotyčnicou na trajektóriu častice, t.j. má smer tangenciálneho vektora.

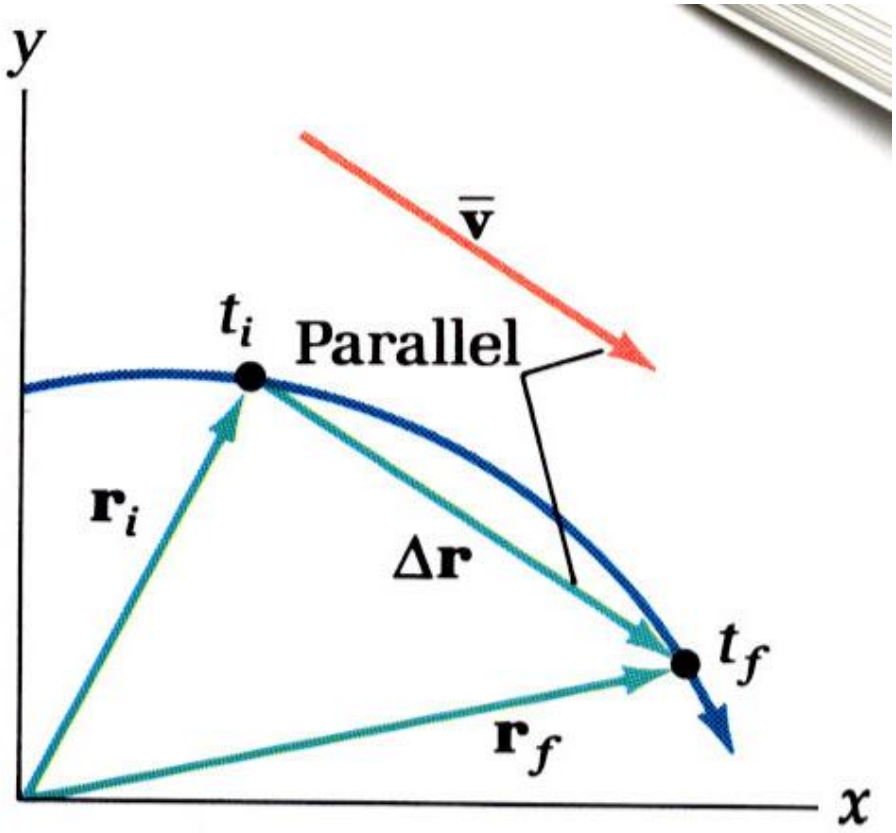
Matematická vsuvka

Vzájomná orientácia vektorov

- Násobenie vektora číslom

$$\vec{b} = s \cdot \vec{a} = \vec{a} \cdot s \quad \begin{cases} s > 0 & \vec{b} \uparrow\uparrow \vec{a} \\ s < 0 & \vec{b} \uparrow\downarrow \vec{a} \end{cases}$$

Pohyb vo viacerých rozmeroch



$$\vec{r}_i = x_i\vec{i} + y_i\vec{j}$$

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j}$$

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$$\Delta\vec{r} = \left[x_f - x_i\right]\vec{i} + \left[y_f - y_i\right]\vec{j} =$$

$$= \Delta x\vec{i} + \Delta y\vec{j}$$

Priemerná rýchlosť

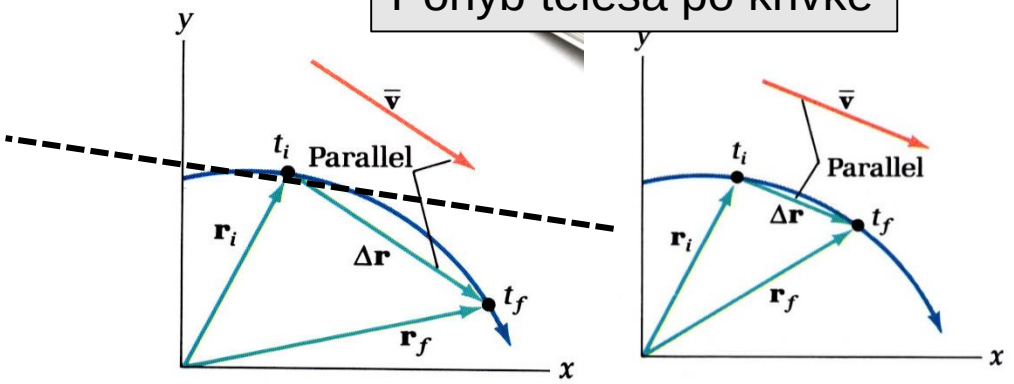
$$\langle\vec{v}\rangle = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\vec{i} + \Delta y\vec{j}}{\Delta t}$$

Okamžitá rýchlosť

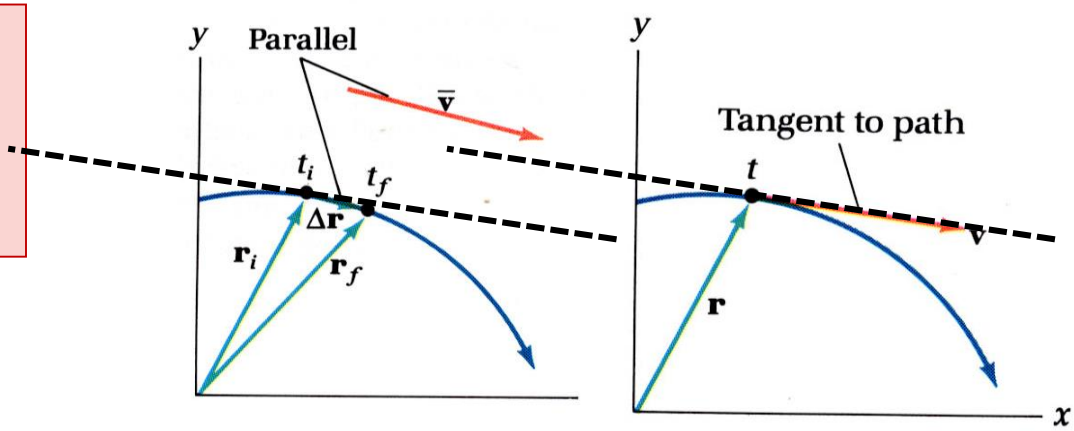
SMER VEKTORA

$$\vec{v} = \frac{d\vec{r}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

Pohyb telesa po krivke

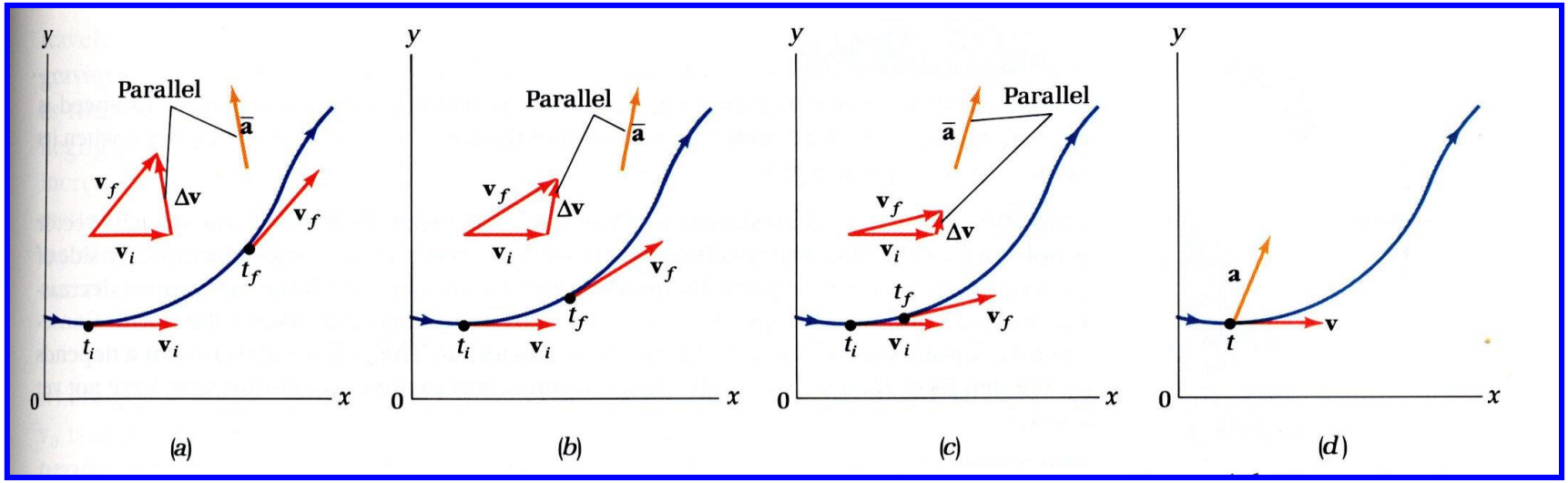


$\Delta\mathbf{r}$ sa **postupne skláňa** ku smeru dotyčnice k trajektórie v bode $\mathbf{r}_i$ t.j. priemerná rýchlosť sa blíži k rýchlosti okamžitej.

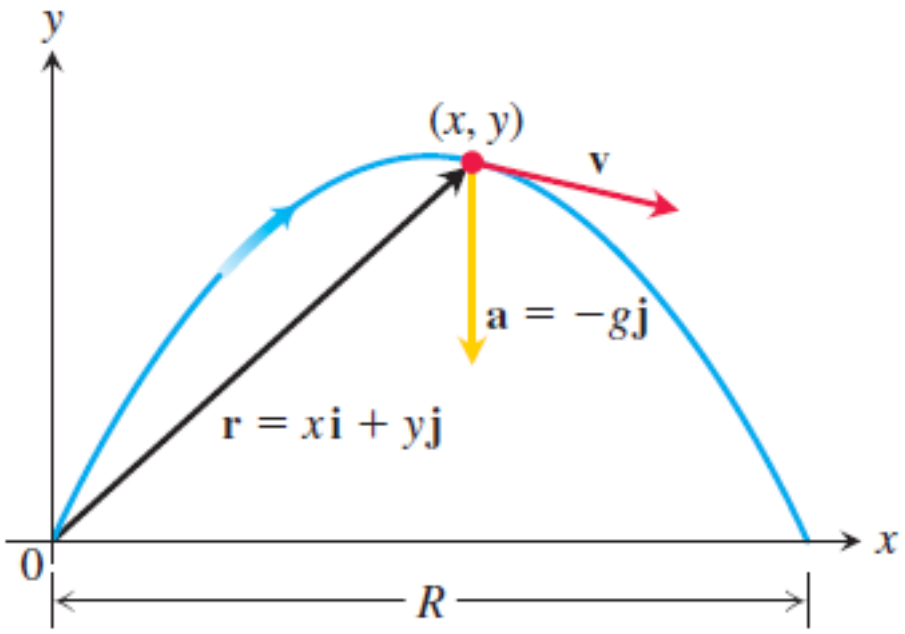


Okamžitá rýchlosť má smer rovnobežný s trajektóriou, t.j. je dotyčnicou k trajektórie.

$$\vec{a} = \frac{d\vec{v}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

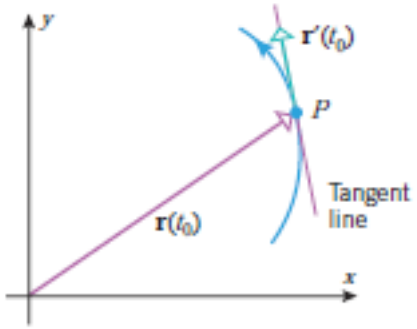


Zrýchlenie telesa nie je vo všeobecnosti dotyčnicou k trajektórii. Zrýchlenie je paralelné k Δv.



Vektor rýchlosti je dotyčnicou na trajektóriu častice, t.j. má smer tangenciálneho vektora.

Derivovanie vektorovej funkcie podľa parametra



$$\vec{r} = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

$$\frac{d\vec{r}}{dt} = \lim_{\Delta t \to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\vec{i} + \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\vec{j} + \lim_{\Delta t \to 0} \frac{\Delta z}{\Delta t}\vec{k} = x'\vec{i} + y'\vec{j} + z'\vec{k}$$

Derivovať vektorovú funkciu podľa parametra znamená derivovať jeho jednotlivé zložky podľa parametra

Pravidlá pre deriváciu vektorovej funkcie podľa parametra

$$\frac{d}{dt}\vec{C} = \vec{0}$$

$$\frac{d}{dt}k\vec{r} = k\frac{d}{dt}\vec{r}$$

$$\frac{d}{dt}\left[\vec{r}_1 \pm \vec{r}_2\right] = \frac{d}{dt}\vec{r}_1 \pm \frac{d}{dt}\vec{r}_2$$

$$\frac{d}{dt}\left[f(t)\vec{r}\right] = f(t)\frac{d}{dt}\vec{r} + \vec{r}\ \frac{d}{dt} f(t)$$

Derivácia vektorových a skalárnych súčinov

$$\frac{d}{dt}\left[\vec{a}\bullet\vec{b}\right]=\lim_{\Delta t\to 0}\frac{\vec{a}\left(t+\Delta t\right)\bullet\vec{b}\left(t+\Delta t\right)-\vec{a}\left(t\right)\bullet\vec{b}\left(t\right)}{\Delta t}=$$

$$=\lim_{\Delta t\to 0}\frac{\vec{a}\left(t+\Delta t\right)\bullet\vec{b}\left(t+\Delta t\right)-\vec{a}\left(t\right)\bullet\vec{b}\left(t+\Delta t\right)}{\Delta t}+\lim_{\Delta t\to 0}\frac{\vec{a}\left(t\right)\bullet\vec{b}\left(t+\Delta t\right)-\vec{a}\left(t\right)\bullet\vec{b}\left(t\right)}{\Delta t}=$$

$$=\lim_{\Delta t\to 0}\frac{\vec{a}\left(t+\Delta t\right)-\vec{a}\left(t\right)}{\Delta t}\bullet\lim_{\Delta t\to 0}\vec{b}\left(t+\Delta t\right)+\lim_{\Delta t\to 0}\vec{a}\left(t\right)\bullet\lim_{\Delta t\to 0}\frac{\vec{b}\left(t+\Delta t\right)-\vec{b}\left(t\right)}{\Delta t}=$$

$$=\frac{d\vec{a}}{dt}\bullet\vec{b}+\vec{a}\bullet\frac{d\vec{b}}{dt}$$

$$\frac{d}{dt}\left[\vec{a}\bullet\vec{b}\right]=\frac{d\vec{a}}{dt}\bullet\vec{b}+\vec{a}\bullet\frac{d\vec{b}}{dt}$$

$$\frac{d}{dt}\left[\vec{a}\times\vec{b}\right]=\frac{d\vec{a}}{dt}\times\vec{b}+\vec{a}\times\frac{d\vec{b}}{dt}$$

Derivovanie konštantnej vektorovej funkcie podľa parametra

$$\vec{r} \bullet \vec{r} = konst \Rightarrow \vec{r} \perp \vec{v}$$

$$\vec{r} \bullet \frac{d\vec{r}}{dt} = 0$$

Jednotkový vektor je kolmý na vektor, ktorý vznikne jeho derivovaním.

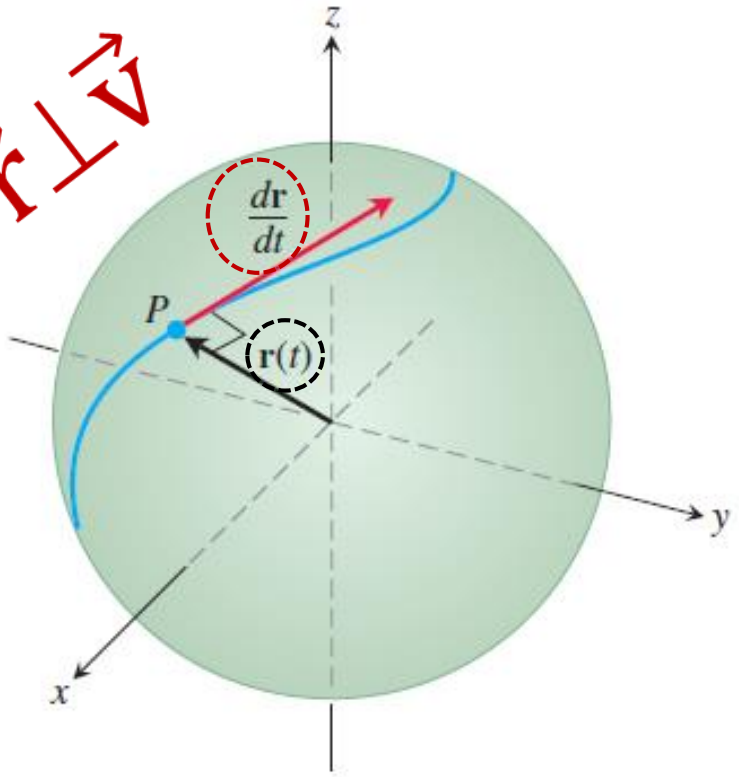
Po zderivovaní vektora podľa parametra, bude tento vektor kolmý na pôvodný vektor ak sa jeho veľkosť nemení

Analogicky : $\left|\vec{v}\right| = konst \Rightarrow \vec{v} \perp \vec{a}$

Vektorové funkcie s konštantnou vzdialenosťou – kinematická interpretácia

Interpretácia na pohybe po guli

$\vec{r} \perp \vec{v}$



Pohyb častice v prípade že veľkosť jej polohového vektora sa nemení t.j. ide po povrchu gule

Vektor rýchlosti je dotyčnicou na trajektóriu častice, t.j. má smer tangenciálneho vektora.

⇨ $\vec{r} \perp \vec{v}$

Príklady

t – skalárny parameter čas

Polohový vektor **r** a vektor rýchlosti **v** sú kolineárne. Ukážte, že kolineárne budú aj všetky derivácie polohového vektora **r**

$$\frac{d\vec{r}}{dt} = \alpha(t)\vec{r}$$

$$\frac{d^2\vec{r}}{dt^2} = \frac{d\alpha(t)}{dt}\vec{r} + \alpha(t)\frac{d\vec{r}}{dt} = \left[\frac{d\alpha}{dt} + \alpha^2\right]\vec{r}$$

Ukážte, že ak **veľkosť vektora | r |** sa **nemení**, potom vektor **rýchlosti je kolmý na polohový vektor**

$$\vec{r} \bullet \vec{r} = konst \Rightarrow \vec{r} \perp \vec{v}$$

$$\vec{r} \bullet \frac{d\vec{r}}{dt} = 0$$

Dôsledky

Jednotkový vektor je kolmý na vektor, ktorý vznikne jeho derivovaním.

Analogicky : $|\vec{v}| = konst \Rightarrow \vec{v} \perp \vec{a}$

Integrovanie

$$\vec{r}(t) = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k}$$

$$\int \vec{r}(t)\,dt = \vec{R}(t) + \vec{C}$$

$$\int_a^b \vec{r}(t)\,dt = \left[\int_a^b f(t)\,dt\right]\vec{i} + \left[\int_a^b g(t)\,dt\right]\vec{j} + \left[\int_a^b h(t)\,dt\right]\vec{k}$$

Integrovanie podľa parametra

Neurčité integrály

$$\int \vec{v}dt = \int \left(3\vec{i} + 2t\vec{j}\right)dt = \left(\int 3dt\right)\vec{i} + \left(\int 2tdt\right)\vec{j} = 3t\vec{i} + \vec{C}_1 + t^2\vec{j} + \vec{C}_2$$
$$= 3t\vec{i} + t^2\vec{j} + \left(\vec{C}_2 + \vec{C}_1\right) = 3t\vec{i} + t^2\vec{j} + \vec{C}$$

KONŠTANTA JE VEKTOR

Určte polohu telesa r(t) v čase t ak jeho rýchlosť $\vec{v} = (3,2t)$ s počiatočnou podmienkou $\vec{r}(1) = (2,5)$

"Prírastok vektora"– vektor posunutia

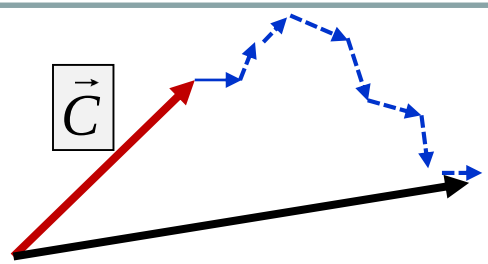
$$\Delta\vec{r} = \int_1^t \vec{v}dt = \left(3t-3\right)\vec{i} + \left(t^2-1\right)\vec{j}$$

Polohový vektor v čase t

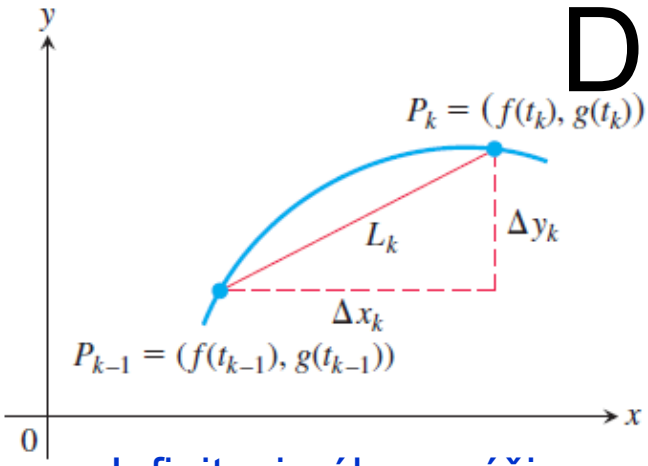
$$\vec{r}\left(1\right) + \Delta\vec{r} = \left(3t-3\right)\vec{i} + \left(t^2-1\right)\vec{j} + 2\vec{i} + 5\vec{j} = \left(3t-1, t^2+4\right)$$

Alternatívny spôsob

$$\mathbf{r}(t) = \int \mathbf{r}'(t)\,dt = \int (3, 2t)\,dt = (3t, t^2) + \mathbf{C}$$
$$\mathbf{r}(1) = (3, 1) + \mathbf{C} = (2, 5)$$



Dĺžka krivky



$$\vec{r} = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

Infinitezimálne zvýšime parameter t a na krivke sa v jednotlivých smeroch posunieme o:

$$x = x(t) \rightarrow dx = \frac{dx}{dt}dt$$

$$y = y(t) \rightarrow dy = \frac{dy}{dt}dt$$

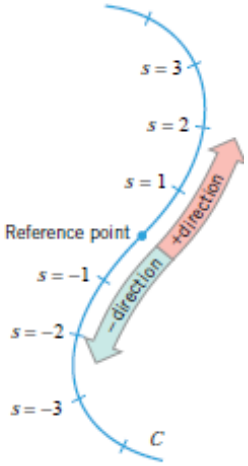
$$z = z(t) \rightarrow dz = \frac{dz}{dt}dt$$

Dráhový, dĺžkový element

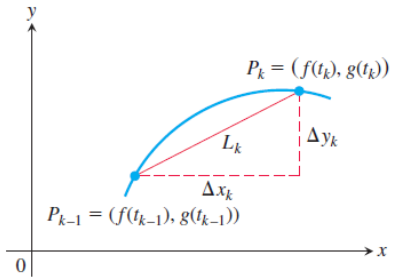
$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt$$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt \quad t \le a \le b$$

Parameter krivky – dĺžka od referenčného bodu



$$\vec{r} = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$



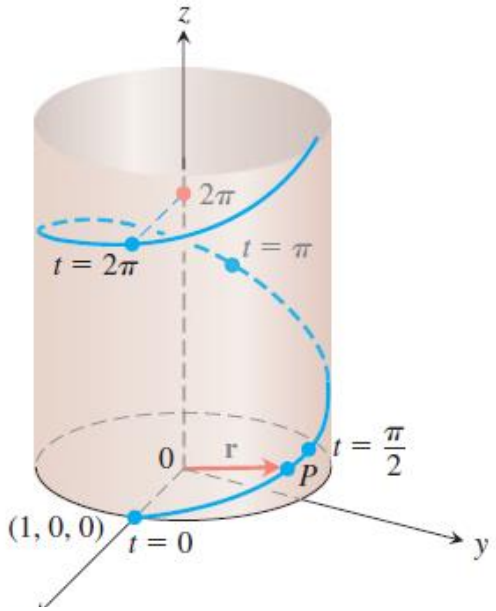
Dráhový, dĺžkový element

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt$$

$$L = \int_{t_0}^{t} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt = \int_{t_0}^{t} \left|\vec{v}(t)\right| dt$$

$$\left|\vec{v}\right| = \frac{ds}{dt}$$

Parameter krivky – dĺžka od referenčného bodu



Nájdite vzdialenosť bodu, ktorý sa nachádza na špirále, v závislosti od parametra t:

$$x = \cos t$$
$$y = \sin t$$
$$z = t$$

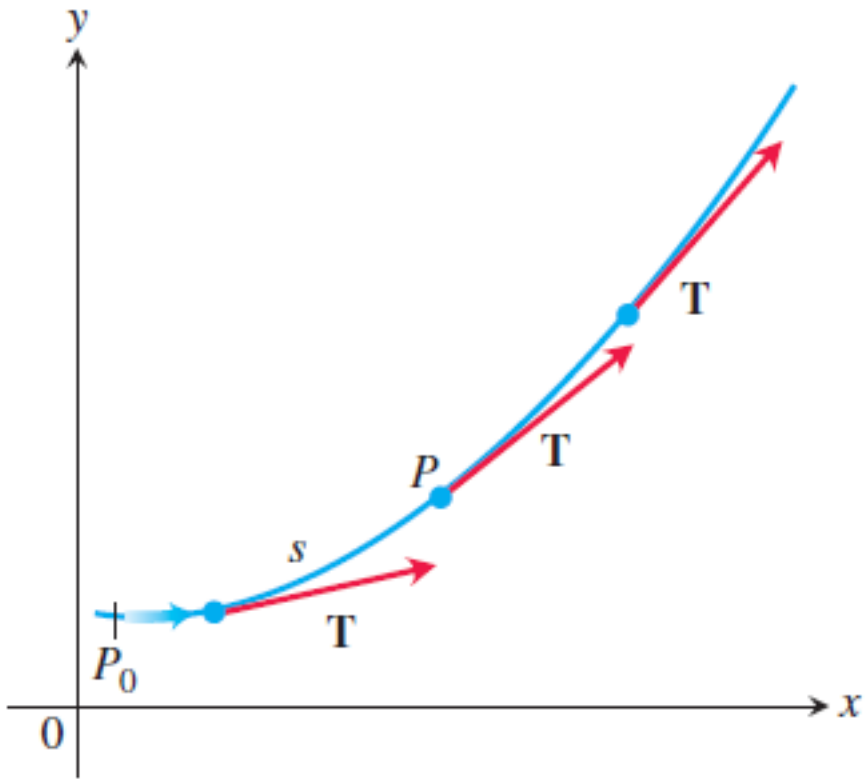
$$s(t) = \int_{t_0}^{t} |\mathbf{v}(\tau)|\, d\tau$$
$$= \int_{0}^{t} \sqrt{2}\, d\tau$$
$$= \sqrt{2}\, t.$$

Vyjadrenie krivky cez inú parametrizáciu:

$$\mathbf{r}(t(s)) = \left(\cos \frac{s}{\sqrt{2}}\right)\mathbf{i} + \left(\sin \frac{s}{\sqrt{2}}\right)\mathbf{j} + \frac{s}{\sqrt{2}}\mathbf{k}.$$

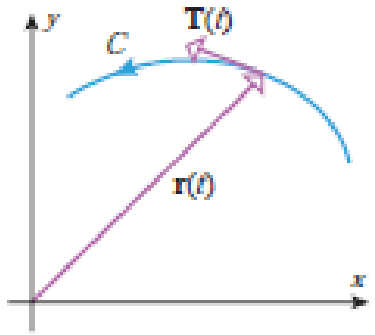
TANGENCIÁLNE A NORMÁLOVÉ JEDNOTKOVÉ VEKTORY

Jednotkový tangenciálny vektor



$$\vec{T}(t) = \frac{d\vec{r}}{dt} \Big/ \left|\frac{d\vec{r}}{dt}\right| = \frac{\vec{v}}{|\vec{v}|}$$

Jednotkový tangenciálny vektor

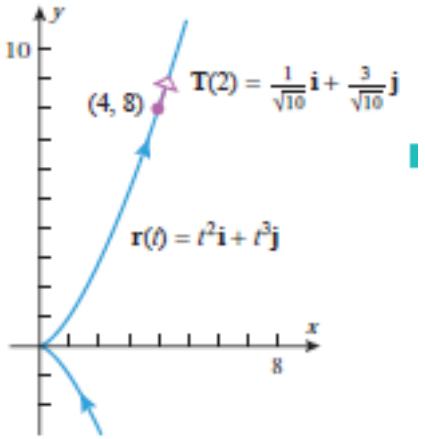


$$\vec{T}(t) = \frac{\vec{r}'}{|\vec{r}'|}$$

Teleso sa pohybuje po krivke $\vec{r}(t) = t^2\vec{i} + t^3\vec{j}$ Nájdite tangenciálny vektor ku krivke v čase t=2s

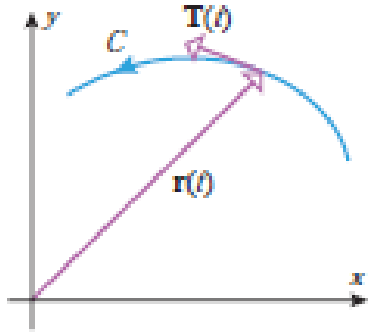
$$\vec{r}(t) = t^2\vec{i} + t^3\vec{j}$$

$$\vec{r}'(t) = 2t\vec{i} + 3t^2\vec{j}$$



$$\mathbf{T}(2) = \frac{\mathbf{r}'(2)}{\|\mathbf{r}'(2)\|} = \frac{4\mathbf{i} + 12\mathbf{j}}{\sqrt{160}} = \frac{4\mathbf{i} + 12\mathbf{j}}{4\sqrt{10}} = \frac{1}{\sqrt{10}}\mathbf{i} + \frac{3}{\sqrt{10}}\mathbf{j}$$

Jednotkový normálový vektor

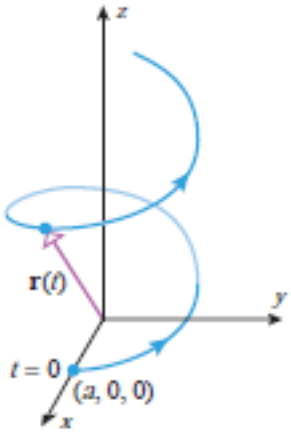


Vektor ktorého veľkosť sa nemení, po zderivovaní podľa parametra je kolmý na pôvodný vektor.

$$\mathbf{N} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$$

$$\vec{N}(t) = \frac{\vec{T}'}{|\vec{T}'|}$$

Nájdite jednotkové tangenciálne a normálové vektory: T a N pre špirálu danú parametricky:



$$\boxed{\begin{aligned} x &= a\cos t \\ y &= a\sin t \\ z &= ct \end{aligned}}$$

$$\mathbf{r}'(t) = (-a\sin t)\mathbf{i} + a\cos t\,\mathbf{j} + c\mathbf{k}$$

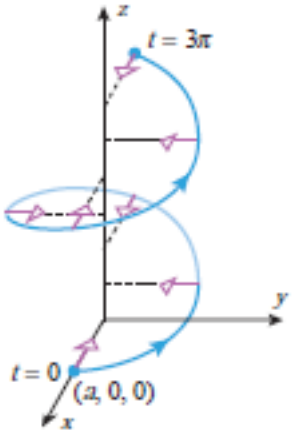
$$\|\mathbf{r}'(t)\| = \sqrt{(-a\sin t)^2 + (a\cos t)^2 + c^2} = \sqrt{a^2 + c^2}$$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} = -\frac{a\sin t}{\sqrt{a^2+c^2}}\mathbf{i} + \frac{a\cos t}{\sqrt{a^2+c^2}}\mathbf{j} + \frac{c}{\sqrt{a^2+c^2}}\mathbf{k}$$

$$\mathbf{T}'(t) = -\frac{a\cos t}{\sqrt{a^2+c^2}}\mathbf{i} - \frac{a\sin t}{\sqrt{a^2+c^2}}\mathbf{j}$$

$$\|\mathbf{T}'(t)\| = \sqrt{\left(-\frac{a\cos t}{\sqrt{a^2+c^2}}\right)^2 + \left(-\frac{a\sin t}{\sqrt{a^2+c^2}}\right)^2} = \sqrt{\frac{a^2}{a^2+c^2}} = \frac{a}{\sqrt{a^2+c^2}}$$

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|} = (-\cos t)\mathbf{i} - (\sin t)\mathbf{j} = -(\cos t\,\mathbf{i} + \sin t\,\mathbf{j})$$

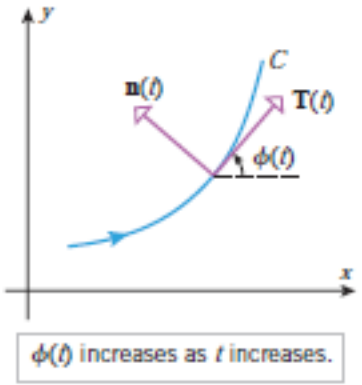


Derivovaním jednotkového vektora NEmusíme dostať jednotkový vektor

Krivka v rovine

Kde vlastne smeruje vektor $\frac{d\vec{T}}{dt}$

$$\vec{T}(t) = \cos\varphi(t)\,\vec{i} + \sin\varphi(t)\,\vec{j}$$



$$\vec{n} = \vec{i}\cos\left[\varphi(t)+\frac{\pi}{2}\right] + \vec{j}\sin\left[\varphi(t)+\frac{\pi}{2}\right] = -\sin\varphi(t)\,\vec{i} + \cos\varphi(t)\,\vec{j}$$

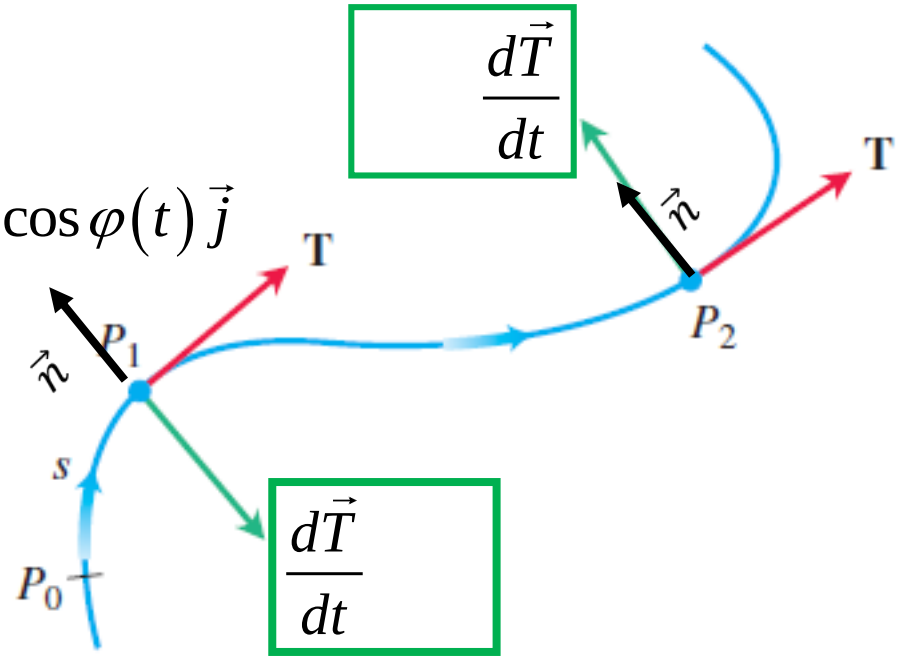
$$\frac{d\vec{T}}{dt} = \frac{d\vec{T}}{d\varphi}\frac{d\varphi}{dt} = \left[-\sin\varphi(t)\,\vec{i} + \cos\varphi(t)\,\vec{j}\right]\frac{d\varphi}{dt} = \vec{n}\frac{d\varphi}{dt}$$

Kde vlastne smeruje $\frac{d\vec{T}}{dt}$

$$\vec{T}(t) = \cos\varphi(t)\,\vec{i} + \sin\varphi(t)\,\vec{j}$$

$$\frac{d\vec{T}}{dt} = \vec{n}\,\frac{d\varphi}{dt}$$

$$\vec{n} = -\sin\varphi(t)\,\vec{i} + \cos\varphi(t)\,\vec{j}$$

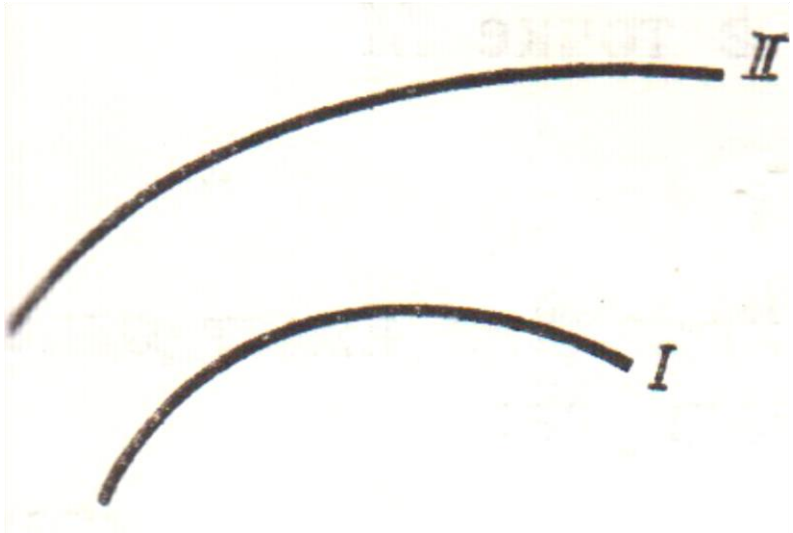


$$\vec{T}(t) = \cos\varphi(t)\,\vec{i} + \sin\varphi(t)\,\vec{j}$$

$$\frac{d\vec{T}}{dt} = \frac{d\vec{T}}{d\varphi}\frac{d\varphi}{dt} = \left[-\sin\varphi(t)\,\vec{i} + \cos\varphi(t)\,\vec{j}\right]\frac{d\varphi}{dt} = \vec{n}\,\frac{d\varphi}{dt}$$

Vektor $\frac{d\vec{T}}{dt}$ smeruje do stredu krivosti krivky

Krivosť

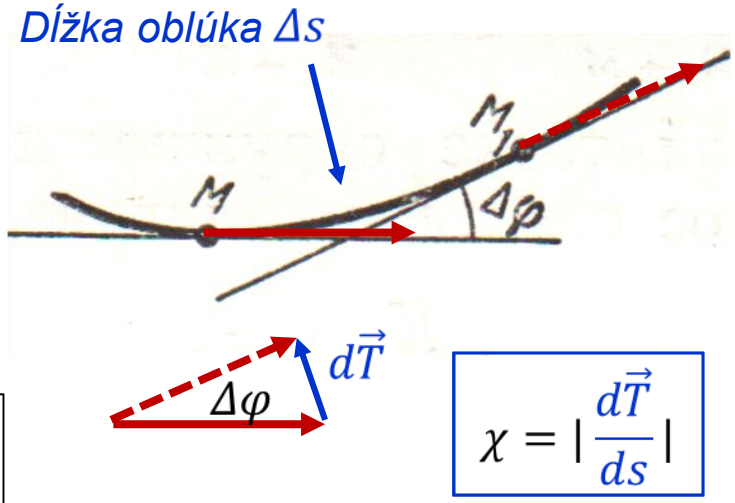


Intuícia – Krivka I je krivšia ako krivka II

Krivosť krivky

$$\chi = \left| \frac{d\vec{T}}{ds} \right|$$

Krivosť krivky určuje veľkosť tohto vektora



$$\chi = \left| \frac{d\vec{T}}{ds} \right|$$

V prípade, že krivka r (t) je parametrizovaná cez parameter t /nie cez dĺžku oblúka/, potom:

$$\chi = \left| \frac{d\vec{T}}{ds} \right| = \left| \frac{d\vec{T}}{dt} \frac{dt}{ds} \right| = \frac{1}{\left| \frac{ds}{dt} \right|} \left| \frac{d\vec{T}}{dt} \right|$$

$\vec{T}$-jednotkový tangenciálny vektor

Toto je veľkosť vektora kolmého na T, t.j. normálového ku krivke, zadefinujme pomocou χ normálový vektor

Pozn. derivácia jednotkového vektora nemusí byť jednotkový vektor

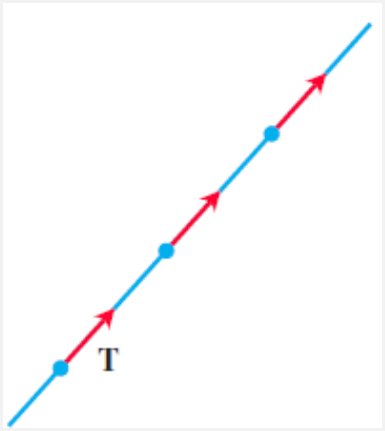
zadefinujme

$$\vec{N} = \frac{1}{\chi} \frac{d\vec{T}}{ds}$$

Krivosť priamky a kružnice

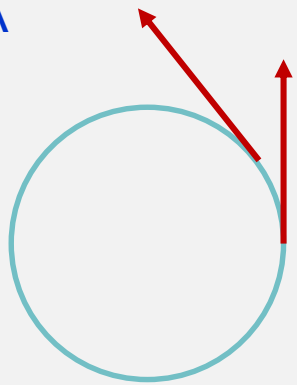
$$\chi = \frac{1}{\left|\frac{ds}{dt}\right|}\left|\frac{d\vec{T}}{dt}\right| = \frac{1}{\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}}\left|\frac{d\vec{T}}{dt}\right|$$

PRIAMKA



$$\chi = \frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right| = 0$$

KRUŽNICA



$$\vec{r} = R\cos t\,\vec{i} + R\sin t\,\vec{j}$$

$$\vec{T} = \frac{d\vec{r}/dt}{|d\vec{r}/dt|} = -\vec{i}\sin t + \vec{j}\cos t$$

$$\frac{d\vec{T}}{dt} = -\vec{i}\cos t - \vec{j}\sin t$$

$$\left|\frac{d\vec{T}}{dt}\right| = 1$$

$$\chi = \frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right| = \frac{1}{R}$$

Normálový vektor

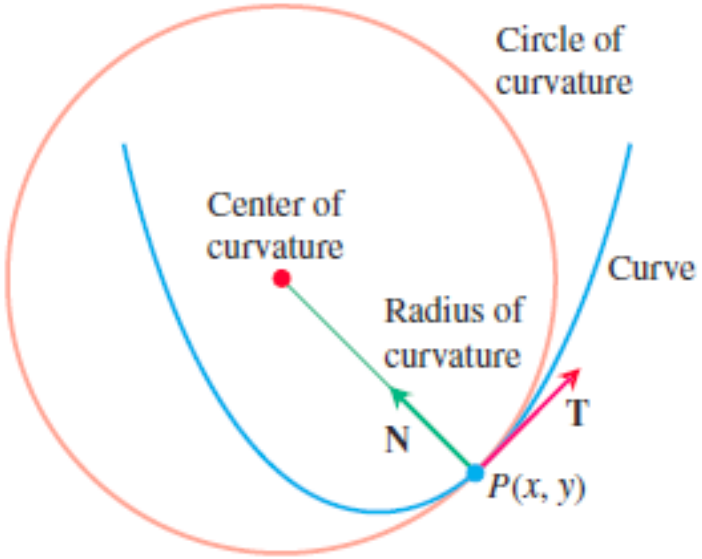
Formula for Calculating N

If $\mathbf{r}(t)$ is a smooth curve, then the principal unit normal is

$$\mathbf{N} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|},$$

where $\mathbf{T} = \mathbf{v}/|\mathbf{v}|$ is the unit tangent vector.

Oskulačná kružnica



Oskulačná kružnica, je kružnica ležiaca v rovine krivky:
1, ktorej dotyčnica v bode P je identická s dotyčnicou ku krivke v tomto bode
2, má rovnaké zakrivenie v bode P ako krivka
3, má stred, ktorý leží vo vnútornej strane oblúka

Zakrivenie krivky

$$\chi = \frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right|$$

Zakrivenie oskulačnej kružnice je $\chi = 1/R$

$$\frac{1}{R} = \frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right|$$

Nájdite graf oskulačnej kružnice k parabole v bode x=0

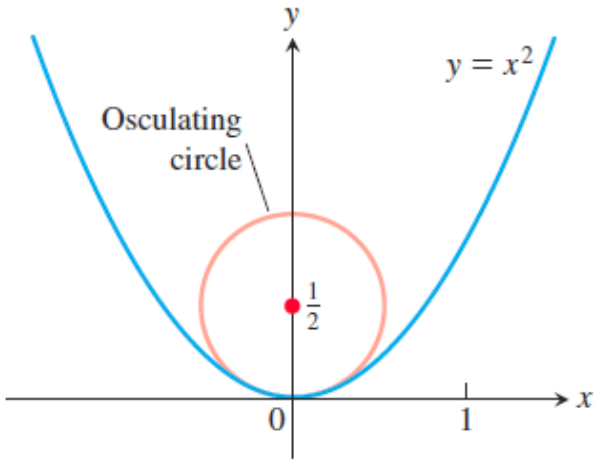
$$\chi = \frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right|$$

$$\vec{r} = t\vec{i} + t^2\vec{j}$$

$$\vec{v} = \vec{i} + 2t\vec{j}$$

$$|\vec{v}| = \sqrt{1+4t^2}$$

$$\vec{T} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{\sqrt{1+4t^2}}\vec{i} + \frac{2t}{\sqrt{1+4t^2}}\vec{j}$$



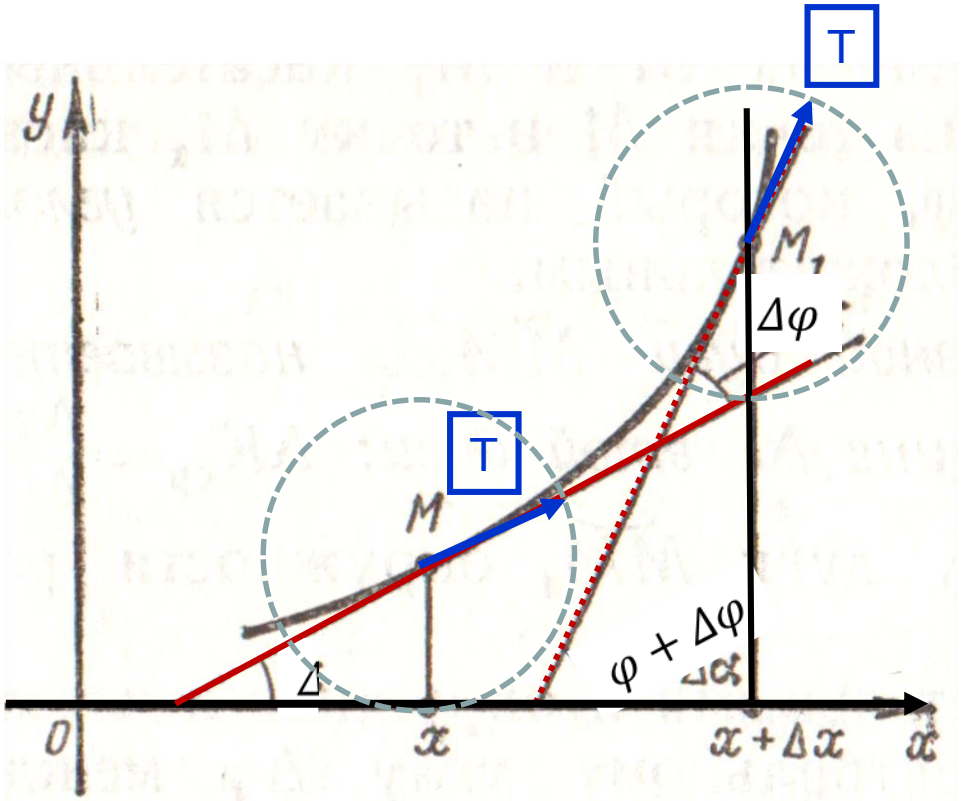
$$\frac{d\,\mathbf{T}}{dt} = -4t(1+4t^2)^{-3/2}\mathbf{i} + [\,2(1+4t^2)^{-1/2} - 8t^2(1+4t^2)^{-3/2}\,]\,\mathbf{j}$$

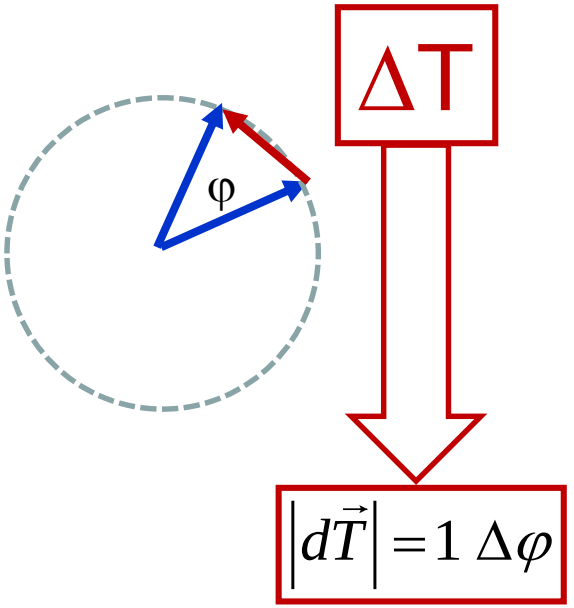
$$\kappa(0) = \frac{1}{|\vec{v}(0)|}\left|\frac{d\vec{T}}{dt}(0)\right| = 2 \quad \Rightarrow$$

$$R(0) = \frac{1}{2}$$

$$\left.\frac{d\vec{T}}{dt}\right|_{t=0} = 2\vec{j} \Rightarrow \vec{N} = \vec{j}$$

Krivosť

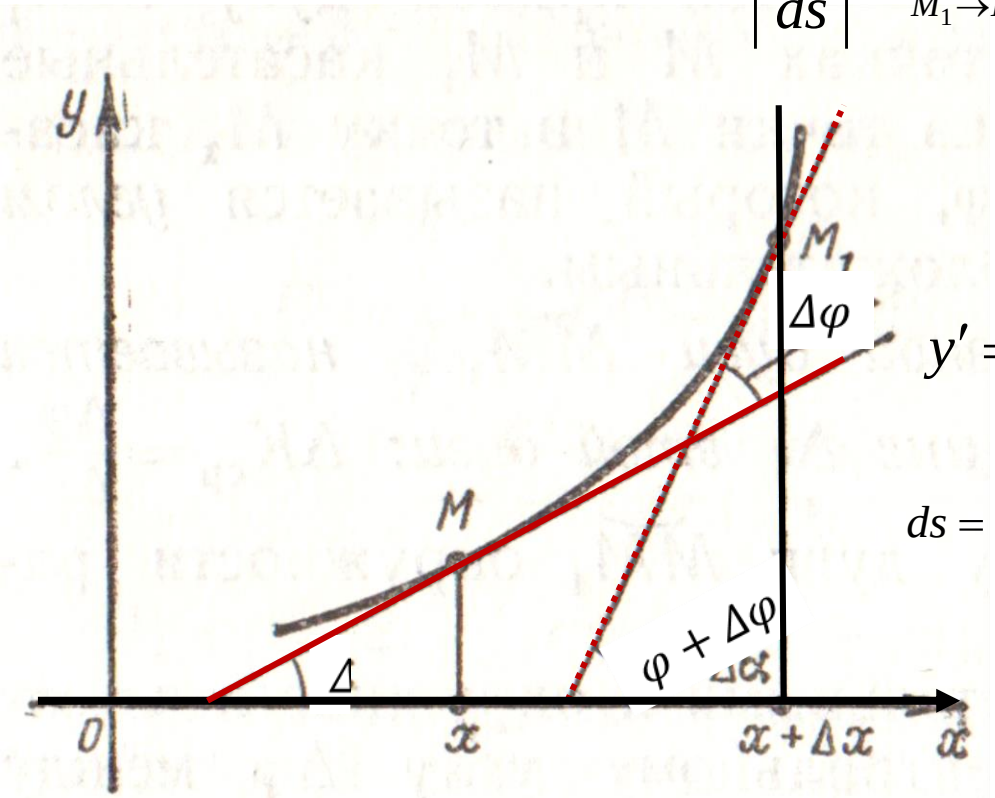




$$\chi = \left|\frac{d\vec{T}}{ds}\right| = \lim_{M_1 \to M} \left|\frac{\Delta\vec{T}}{\Delta s}\right| = \lim_{M_1 \to M} \frac{\left|\Delta\vec{T}\right|}{|\Delta s|} = \lim_{\Delta x \to 0} \left|\frac{1\,\Delta\varphi}{\Delta s}\right| = \frac{\lim\limits_{\Delta x \to 0} \left|\frac{\Delta\varphi}{\Delta x}\right|}{\lim\limits_{\Delta x \to 0} \left|\frac{\Delta s}{\Delta x}\right|} = \frac{\left|\frac{d\varphi}{dx}\right|}{\left|\frac{ds}{dx}\right|}$$

Krivosť

$$\chi = \left|\frac{d\vec{T}}{ds}\right| = \lim_{M_1 \to M}\left|\frac{\Delta\vec{T}}{\Delta s}\right| = \lim_{\Delta x \to 0}\left|\frac{\Delta\varphi}{\Delta s}\right| = \frac{\lim\limits_{\Delta x \to 0}\left|\frac{\Delta\varphi}{\Delta x}\right|}{\lim\limits_{\Delta x \to 0}\left|\frac{\Delta s}{\Delta x}\right|} = \frac{\left|\frac{d\varphi}{dx}\right|}{\left|\frac{ds}{dx}\right|}$$



$$y' = tg\varphi \quad \Rightarrow \frac{d\varphi}{dx} = \left(arctg\, y'\right)' = \frac{y''}{1+y'^2}$$

$$ds = \sqrt{(dx)^2 + (dy)^2} = dx\sqrt{1+y'^2} \quad \Rightarrow \frac{ds}{dx} = \sqrt{1+y'^2}$$

$$\chi = \left|\frac{\frac{d\varphi}{dx}}{\frac{ds}{dx}}\right| = \frac{\left|\frac{y''}{1+y'^2}\right|}{\left|\sqrt{1+y'^2}\right|}$$

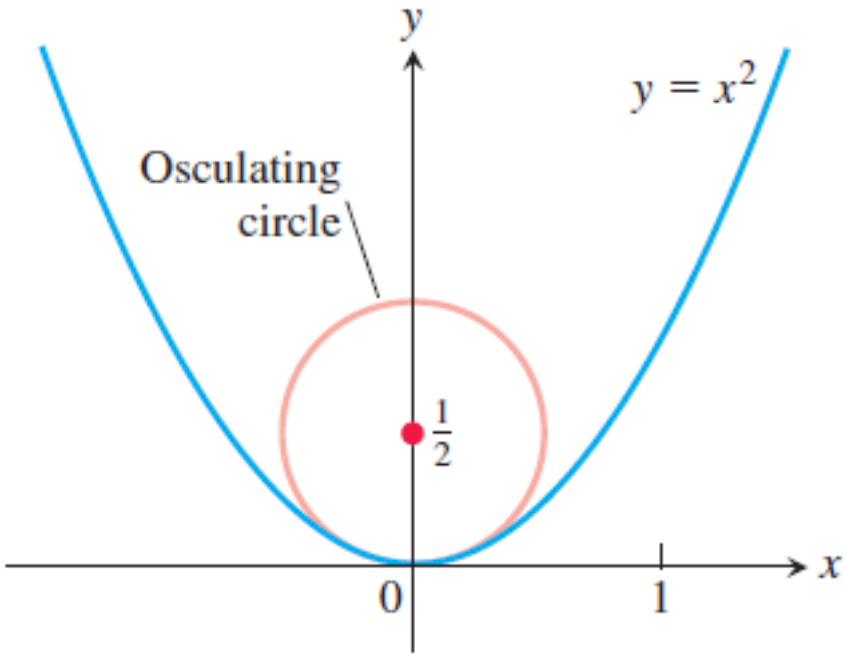
$$\chi = \frac{|y''|}{\left(1+y'^2\right)^{3/2}}$$

Nájdite graf oskulačnej kružnice pre parabolu $y=x^2$ v bode $x=0$

$$y = x^2$$

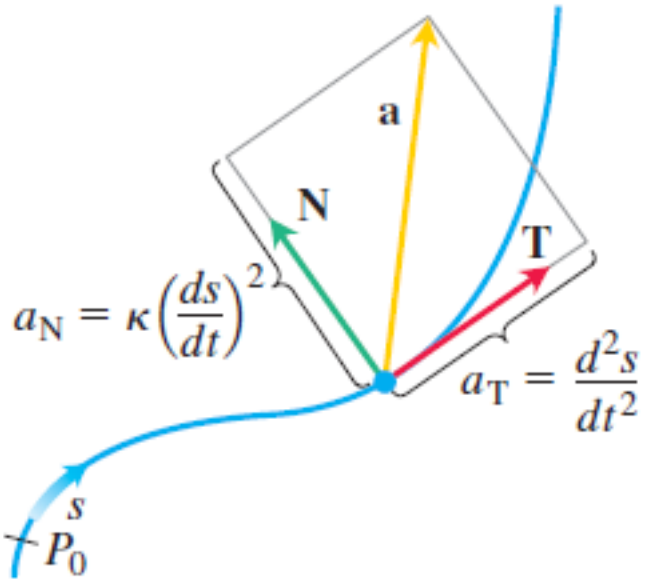
$$\Rightarrow$$

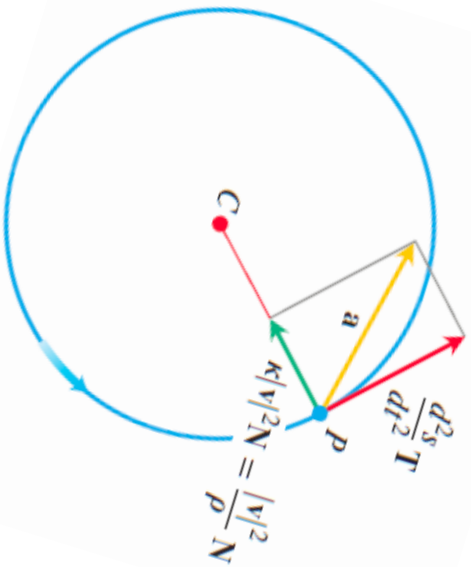
$$\chi = \frac{|y''|}{\left(1+y'^2\right)^{3/2}}$$



Vedeli by sme nájsť vo všeobecnosti stred oskulačnej kružnice ?

Tangenciálne a normálové zrýchlenie





$$\vec{v} = v\vec{T} \qquad v = ds/dt$$ – veľkosť rýchlosti

$$\frac{d\vec{T}}{ds} = \chi\vec{N}$$

$$\frac{d}{dt}\vec{v} = \frac{dv}{dt}\vec{T} + v\frac{d\vec{T}}{dt} = \frac{dv}{dt}\vec{T} + v\frac{d\vec{T}}{ds}\frac{ds}{dt} = \frac{dv}{dt}\vec{T} + v^2\chi\vec{N} = \boxed{\frac{dv}{dt}}\vec{T} + \frac{v^2}{R}\vec{N}$$

Tangenciálne zrýchlenie – derivácia veľkosti rýchlosti podľa času

Bez nájdenia vektorov N a T rozložte zrýchlenie do týchto smerov

Pohyb začal v čase t=0s.

$$\vec{r} = \left(\cos t + t\sin t\right)\vec{i} + \left(\sin t - t\cos t\right)\vec{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \left(-\sin t + \sin t + t\cos t\right)\vec{i} + \left(\cos t - \cos t + t\sin t\right)\vec{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \left(\cos t - t\sin t\right)\vec{i} + \left(\sin t + t\cos t\right)\vec{j}$$

$$\left|\vec{v}\right| = \sqrt{t^2} = \left|t\right| = t \quad t \geq 0$$

$$a_t = \frac{d\left|\vec{v}\right|}{dt} = 1$$

$$\left|\vec{a}\right| = t^2 + 1$$

$$a_n = \sqrt{a^2 - a_t^2} = \sqrt{t^2} = t$$

$$\boxed{\vec{a} = \vec{T} + t\vec{N}}$$