Diffusion:
a new approach to recommender systems

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The playground

- available data:
  - ratings from M users for N movies
  - integer scale: 1 (very bad), 2, 3, 4, 5 (perfect)

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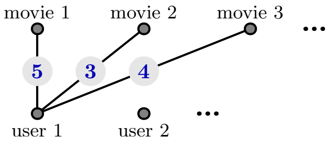
  - ratings from M users for N movies
  - integer scale: 1 (very bad), 2, 3, 4, 5 (perfect)
- both M and N can be very large
  - thus we need an efficient recommendation method
- prediction methods already available:
  - **average rating**
    (recommendation by overall “quality”)
  - **correlation based**
    (recommendation by the user-user similarity)
  - . . .

Representations of the data

- available ratings: weighted bipartite graph



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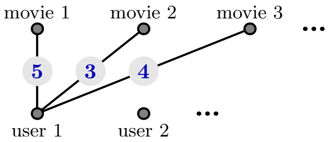
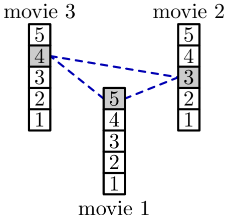
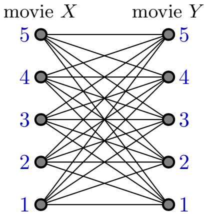


- movie-to-movie projection of the data



Diffusion in the movie-to-movie network

- opinions expressed by a particular user spread over the network



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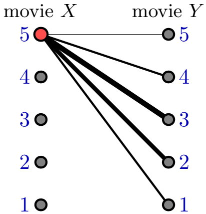

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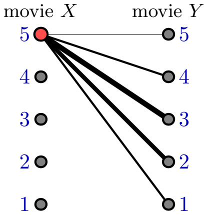

- summary:
  - **key process**: diffusion on a complex network
  - **underlying network**: movie-to-movie projection
  - **boundary condition**: ratings given by one user

Mathematical formalism

- the rating of user *i* for movie α we represent as a 5-dimensional vector

$$1 \rightarrow \boldsymbol{v}_{i\alpha} = \{1, 0, 0, 0, 0\}^{\mathsf{T}}, \ldots, \; 5 \rightarrow \boldsymbol{v}_{i\alpha} = \{0, 0, 0, 0, 1\}^{\mathsf{T}}$$

- the link between movies α and β is a 5×5 matrix

$$\mathsf{W}_{\alpha\beta} = \sum_{i=1}^{M} \lambda_i \boldsymbol{v}_{i\alpha} \boldsymbol{v}_{i\beta}^{\mathsf{T}}$$

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  - for a user who has rated k_i movies we use $\lambda_i = 1/(k_i - 1)$

The diffusion process

- matrices $W_{\alpha\beta}$ form the overall symmetric matrix W ($5N \times 5N$)
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- by the column normalization of W we obtain Ω which describes the diffusion process on the movie-to-movie network
- Ω acts on the $5N$-dimensional "state" vector $\boldsymbol{h}$
  - elements 1-5 correspond to movie 1
  - elements 6-10 correspond to movie 2
  - ...

Boundary conditions

- for a given user we set $\boldsymbol{h}_0$ according to the expressed ratings

$$\boldsymbol{h}_0 = \{\underbrace{1,0,0,0,0}_{\text{1 for movie 1}},\underbrace{0,0,0,1,0}_{\text{4 for movie 2}},\dots\}^{\mathsf{T}}.$$

  - if the rating has not been given yet, we set only zeros for this movie

- then we iterate using the equation

$$\boldsymbol{h}_{n+1} = \hat{\Omega}\boldsymbol{h}_n$$

  - $\hat{\Omega}$ differs from Ω in the fact that it **preserves expressed opinions** of the given user

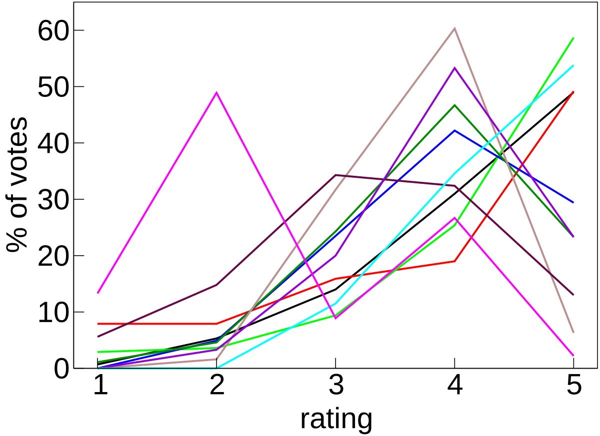
Interpretation of the results

- after $n_{\max}$ iterations we stop
- e.g. for the movie X (not rated yet by the given user) we obtain $\{0.1, 0.1, 0.4, 0.3, 0.0\}^{\mathrm{T}}$
- the standard weighted average transforms to a scalar value

$$\hat{v} = \frac{0.1 \times 1 + 0.1 \times 2 + 0.4 \times 3 + 0.3 \times 4}{0.1 + 0.1 + 0.3 + 0.4} = \mathbf{3.0}$$

  - this is the prediction for movie X

The polarization problem



Solution to the polarization problem

- we unify the ratings by the linear transformation

$$v_{i\alpha} \rightarrow M_i + (v_{i\alpha} - \mu_i)\,\frac{S_i}{\sigma_i}$$

  - μ_i and σ_i is the avearage rating and the standard deviation of ratings for user i
  - M_i and S_i is the avearage rating and the standard deviation of ratings for the whole society
(only movies rated by user i taken into account)

Summary of the algorithm

$W_{\alpha\beta}$

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$$\mathrm{W}_{\alpha\beta}$$
$$\downarrow$$
$$\mathrm{W}$$

Summary of the algorithm

$$W_{\alpha\beta}$$
$$\downarrow$$
$$W$$
$$\downarrow$$
$$\Omega$$

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$$\mathsf{W}_{\alpha\beta}$$
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Summary of the algorithm

unification

$$\downarrow$$

$$W_{\alpha\beta}$$

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$$\downarrow$$

repersonalization

Numerical tests

- data of GroupLens project (`www.grouplens.org`)
    - 943 users, 1682 movies, 100 000 ratings
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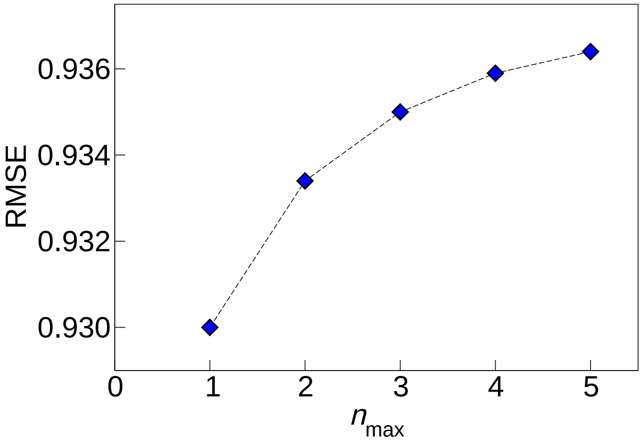
  - sparsity around 6%
- randomly chosen $n = 10\,000$ ratings we move to probe $\mathcal{P}$
- the remaining 90 000 ratings we use to predict the deleted
- two standard error measures of the prediction performance

$$\mathrm{RMSE} = \left[\frac{1}{n}\sum_{(i,\alpha)\in\mathcal{P}}(v_{i\alpha} - \hat{v}_{i\alpha})^2\right]^{1/2}$$

$$\mathrm{MAE} = \frac{1}{n}\sum_{(i,\alpha)\in\mathcal{P}}|v_{i\alpha} - \hat{v}_{i\alpha}|$$

Performance against the number of iterations



Comparison with standard methods

RMSE values for the tested methods

method	no unification	with unification
movie average	1.18	1.01
correlation-based	1.09	1.09
diffusion-based	1.00	0.93

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  - substraction of the user average μ_i is a part of correlation-based methods and thus unification does not improve performance here
  - the diffusion-based method outperforms the other two

Thank You for Your attention